Jacob 1946: ASCE "Proceedings (Papers), pp. 629-646
Jacob 1947: ASCE "Transactions", Paper no. 2321, pp. 1047-1064 [discussions pp.1065-1070]

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2321

DRAWDOWN TEST TO DETERMINE EFFECTIVE RADIUS OF ARTESIAN WELL

BY C. E. JACOB,[1] ASSOC. M. ASCE

WITH DISCUSSION BY MESSRS. N. S. BOULTON, CARL ROHWER, R. M. LEGGETTE, M. R. LEWIS, AND C. E. JACOB.

SYNOPSIS

The drawdown in an artesian well that is pumped has two components: The first, arising from the "resistance" of the formation, is proportional to the discharge; and the second, termed "well loss" and representing the loss of head that accompanies the flow through the screen and upward inside the casing to the pump intake, is proportional approximately to the square of the discharge. The resistance of an extensive artesian bed increases with time as the ever-widening area of influence of the well expands. Consequently, the specific capacity of the well, which is discharge per unit drawdown, decreases both with time and with discharge.

The multiple-step drawdown test outlined in this paper permits the determination of the well loss and of the "effective radius" of the well. The trend of drawdown is observed in the pumped well and in one or more near-by observation wells as the discharge is increased in stepwise fashion. A simple graphical procedure gives the permeability and the compressibility of the bed. From these several factors it is possible to predict the pumping level at any time for any given discharge.

NOTATION

The letter symbols in this paper are defined where they first appear and are assembled alphabetically, for convenience of reference, in the Appendix.

INTRODUCTION

It has long been known that the discharge of an artesian well is almost, but not quite, proportional to the drawdown. In a well that is pumped the drawdown is the difference between the static water level and the pumping

NOTE.—Published in May, 1946, *Proceedings*. Positions and titles given are those in effect when paper or discussion was received for publication.

[1] Hydr. Engr., Water Resources Branch, U. S. Geological Survey, Washington, D. C.

water lev..., customarily measured after several hours of continuous operation. Usually the major part of this loss of head, or drawdown, occurs in the formation, where the energy expended in overcoming the frictional resistance of the sand against the slowly moving water is directly proportional to that rate of motion. A smaller although no less important part of the loss of head occurs as the water moves at relatively high velocities through the screen and upward inside the casing to the intake of the pump. This head loss is approximately proportional to some higher power of the velocity approaching the square of the velocity. Adding these two components of drawdown:

$$s_w = B\,Q + C\,Q^2 \qquad (1)$$

—approximately. Considering the drawdown, s_w, analogous to electric potential drop and the discharge, Q, analogous to electric current, the factor B can be defined as the "resistance" of the formation. This factor represents the total hydraulic resistance of the formation, from the face of the well to some distance where the head drop is virtually zero and where the radial motion of water toward the discharging well has not yet begun. The ratio of discharge to drawdown, called "specific capacity," is seen from Eq. 1 to be

$$Q/s_w = 1/(B + C\,Q) \qquad (2)$$

Clearly, then, the specific capacity must vary, however slightly, with the discharge. Also, it must vary with time because, as will be shown, the resistance B increases with time as the ever-widening area of influence of the well expands.

It is the purpose of this paper to demonstrate that the factors B and C can be determined by a procedure that is little more elaborate than the usual "drawdown test" made to determine the specific capacity and to check the performance of the pump and motor. This is accomplished simply by controlling, more closely, the stepwise variation of the discharge and by observing, more frequently and more accurately, the trend of the pumping level as it is lowered.

Distribution of Drawdown in and Near an Artesian Well

Fig. 1 shows three typical examples of artesian wells that completely penetrate extensive formations of assumedly homogeneous structure and uniform thickness. Fig. 1(*a*) is the ideal case of an uncased hole drilled through a water-bearing sandstone confined above and below by impervious shales. Virtually all the head loss occurs in the formation, since no well screen or slotted casing is present to impede the flow of water into the hole There must inevitably be some additional friction as the water moves up the hole, and consequently the pumping water level in the hole does not coincide exactly with the head at the face of the hole just inside the formation but rather stands somewhat lower, perhaps as indicated by the dashed line in Fig. 1(*a*).

Fig. 1(*b*) shows a common type of construction in an unconsolidated artesian sand, where a screen is necessary. For comparison, this sand is shown as having the same thickness and permeability as the sandstone of Fig. 1(*a*)

In the second case, with the same discharge there is the same drop in head within the formation; but in addition there is a "well loss" which includes the head lost through friction as the water flows upward inside the screen and casing to the pump intake as well as to the head drop across the screen. The radial distribution of head within the formation is about the same in both

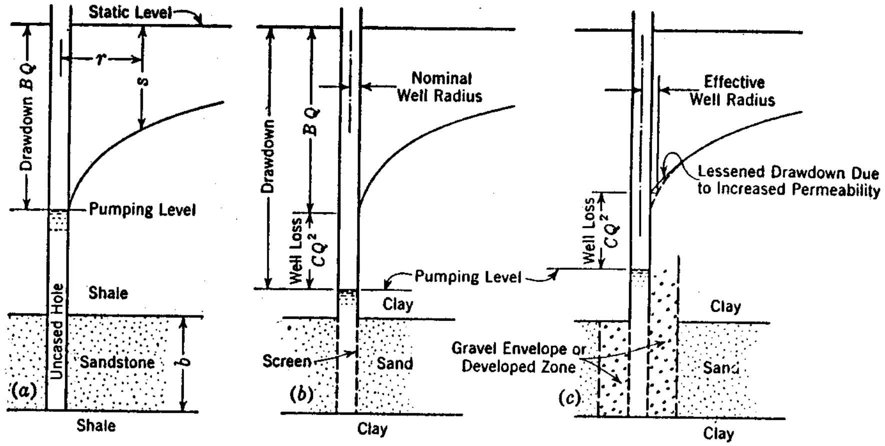


Fig. 1.—Typical Examples of Artesian Wells, Showing Distribution of Drawdown

cases, which may indicate that the sand in Fig. 1(*b*) is not greatly affected by drilling or developing operations; the screen effectively retains all the sand including fines.

A common, although not always essential, item of artesian well construction in unconsolidated formations—the gravel envelope—is shown in Fig. 1(*c*). The gravel envelope is particularly effective when the water-bearing sand is fine and of uniform grading. When properly constructed, it is also useful in other situations—to prevent the fines from being drawn into the well. If the size of gravel is properly chosen, the head loss in the immediate vicinity of the screen is reduced to less than it would be if the natural undisturbed water-bearing formation which the gravel replaced were there. Developing a well to remove the fines from the material surrounding the screen has a similar effect. In some sands, developing operations alone are adequate and gravel-wall construction is not needed. In either case the increased permeability of the material surrounding the well lessens the drawdown and increases the effective radius of the well. "Effective radius" is defined as that distance, measured radially from the axis of the well, at which the theoretical drawdown based on the logarithmic head distribution (defined subsequently by Eq. 4) equals the actual drawdown just outside the screen (see Fig. 1(*c*)).

The dashed curved line in Fig. 1(*c*) represents the head distribution that would exist if the water-producing bed were left in place undisturbed, with uniform permeability. That curve duplicates the drawdown curves in Figs. 1(*a*) and 1(*b*). It is so shaped because, to maintain a steady flow of water (at the rate Q) toward the well, the hydraulic gradient must be inversely proportional to the radial distance; or, actually:

$$\frac{ds}{dr} = -\frac{Q}{2\pi k b r} \quad \ldots\ldots(3)$$

in which b is the thickness of the bed and k is the "permeability" or transmission constant of the sand, defined as:

> "* * * the quantity [volume] of water that would be transmitted in unit time through a cylinder of the soil of unit length and unit cross-section under unit difference in head at the ends."[2]

Integrating Eq. 3 between the fixed limit r_w and the variable limit r:

$$s_w - s = \frac{Q}{2\pi T}\log_e\frac{r}{r_w} \quad \ldots\ldots(4)$$

In Eq. 4, the "transmissibility," T, is the product of k and b.[3] Eq. 4 gives a logarithmic distribution of drawdown that holds in the immediate vicinity of a well pumping from an artesian bed which it penetrates completely. Assuming that the drawdown, s_w, is known at the effective well radius, r_w, the drawdown at some greater distance may be determined easily. Actually, as a matter of common knowledge, the drawdown in an artesian well increases continuously with time (rapidly at first, of course, and then more slowly) as long as the discharge continues at a steady rate and also provided that the well is not too near the margin of the aquifer where the head may be maintained essentially constant despite the withdrawal of water. To determine the drawdown at the well and its distribution throughout an extensive aquifer at any time, it is necessary to study the flow of the confined water in response to varying heads more closely, taking into consideration the compressibility of the water and also the compressibility of the sand bed.

Theory of Nonsteady Radial Flow in an Extensive Artesian Aquifer

Consider a cylindrical shell of height b, inner radius r, and outer radius $(r + \delta r)$ concentric with the axis of the well. By the principle of continuity, the net outward flow of water from this shell must equal the time rate of decrease of the volume of water within the shell, referred to a constant (atmospheric) pressure. The total volume of water in the shell is

$$V_w = 2\pi r\,\delta r\,b\,n \quad \ldots\ldots(5)$$

in which n is the porosity of the sand. The time rate of decrease of this volume is $2\pi r\,\delta r\,b\,n\,\beta\,\frac{\partial(\gamma s)}{\partial t}$, in which γ is the specific weight of the water and β is its compressibility. To allow for the compressibility of the water-producing bed, which is assumed to be compacted elastically as the pressure is reduced

[2] "Theoretical Investigation of the Motion of Ground Waters," by Charles S. Slichter, *Nineteenth Annual Report*, U. S. Geological Survey, 1899, Pt. II, p. 323.

[3] "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage," by Charles V. Theis, *Transactions*, Am. Geophysical Union, Pt. II, 1935, p. 520.

and as the water is allowed to expand, an apparent compressibility, β', is substituted for β. Experience[4,5] indicates β' to be several times the actual water compressibility, β. Combining several factors into a nondimensional coefficient,

$$S = \gamma\,\beta'\,b\,n \quad \ldots\ldots(6a)$$

and $2\pi r\,\delta r\,S\,\frac{\partial s}{\partial t}$ is the time rate of decrease of the volume of water. In Eq. 6a S is the "coefficient of storage,"[6,7] which defines the volume of water that a unit decline in head releases from storage in a vertical prism of the aquifer of unit cross-sectional area.

The apparent fluid compressibility, β', is related to the respective compressibilities of the water and the sand as follows:

$$\beta' = \beta + \frac{\alpha}{n} \quad \ldots\ldots(6b)$$

Again β is the compressibility of the water, and n is the porosity of the sand. The symbol α represents the "compressibility" of the sand bed—that is, the relative decrease in thickness of the bed per unit increase of the vertical component of compressive stress in the sand.

The foregoing relation in somewhat different notation was derived in an earlier paper[8] by the writer in which the theory of nonsteady flow in elastic artesian aquifers was developed.

The outward flow of water from the shell through its inner cylindrical surface is equal to $-2\pi r\,T\left(\frac{\partial s}{\partial r}\right)$. Similarly, the outward flow through the outer cylindrical surface is $2\pi\,T\,(r + \delta r)\left(\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\delta r\right)$. Assuming the upper and lower bounding planes of the aquifer to be impermeable, the sum of these two terms may be equated to the time rate of decrease of the enclosed volume of water. Expanding, eliminating differentials of higher order, simplifying, and dividing through by $(2\pi r\,T\,\delta r)$:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t} \quad \ldots\ldots(7)$$

The solution of this fundamental differential equation that is sought here must satisfy the following conditions:

$$s = 0 \quad \text{for} \quad t \leqq 0 \quad \ldots\ldots(8a)$$

$$\lim_{r \to \infty} s = 0 \quad \text{for} \quad t > 0 \quad \ldots\ldots(8b)$$

[4] "Notes on the Elasticity of the Lloyd Sand on Long Island, New York," by C. E. Jacob, *Transactions*, Am. Geophysical Union, 1941, Pt. III, pp. 783–787.

[5] "Application of Coefficients of Transmissibility and Storage to Regional Problems in the Houston District, Texas," by W. F. Guyton, *ibid.*, pp. 756–770.

[6] "The Significance and Nature of the Cone of Depression in Ground-Water Bodies," by Charles V. Theis, *Economic Geology*, 1938, p. 894.

[7] "The Source of Water Derived from Wells," by Charles V. Theis, *Civil Engineering*, May, 1940, p. 277.

[8] "On the Flow of Water in an Elastic Artesian Aquifer," by C. E. Jacob, *Transactions*, Am. Geophysical Union, 1940, Pt. II, pp. 574–586.

and

$$\operatorname*{Limit}_{r \to 0} \left(r \frac{\partial s}{\partial r} \right) = - \frac{Q}{2 \pi T} \quad \text{for} \quad t > 0 \ldots\ldots\ldots\ldots (8c)$$

The answer is given in terms of an infinite series[8,9,10] as follows:

$$s = \frac{Q}{4 \pi T} \left(- 0.5772 - \log_e u + u - \frac{u^2}{2 \times 2!} - \frac{u^3}{3 \times 3!} + \cdots \right) \ldots (9)$$

in which

$$u = \frac{t^*}{t} = \frac{r^2 S}{4 T t}, \ldots\ldots\ldots\ldots (10)$$

In still simpler notation,

$$s = \frac{Q}{4 \pi T} W(u) \ldots\ldots\ldots\ldots (11)$$

Fig. 2 gives a nondimensional plotting of Eq. 9 or of Eq. 11. The well starts pumping at a steady rate Q at time zero ($t/t^* = 0$). The drawdown at a given distance from the well increases very slowly at first and reaches a

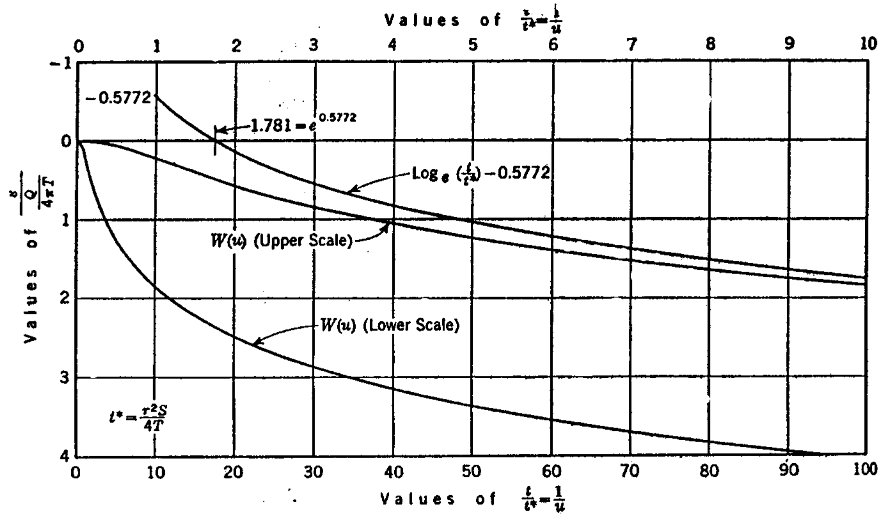


FIG. 2.—NONDIMENSIONAL TIME-DRAWDOWN CURVES, EXACT AND APPROXIMATE, FOR SINGLE WELL DISCHARGE AT A STEADY RATE, FROM AN EXTENSIVE ARTESIAN AQUIFER

maximum rate of increase at $t/t^* = 1$. As this is the point of inflection on the time-drawdown curve, t^* may be called the "inflectional time." Thereafter the rate of increase of drawdown diminishes continually but never vanishes. Theoretically, the drawdown becomes infinite at infinite time.

In Fig. 3 the same equation is plotted on semilogarithmic paper, again in nondimensional form. For sufficiently large values of t, the W-function may be approximated by a simple logarithmic expression, that plots as a straight line on that graph. Thus, the drawdown after sufficient time has elapsed is given approximately by

$$s = \frac{Q}{4 \pi T} \left(\log_e \frac{t}{t^*} - 0.5772 \right) \ldots\ldots\ldots\ldots (12)$$

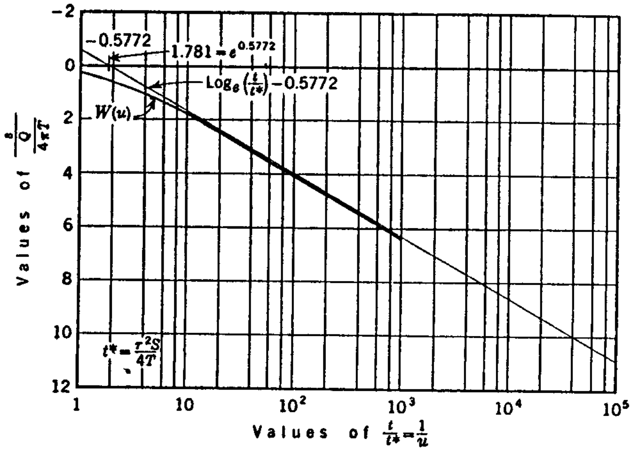


FIG. 3.—SEMILOGARITHMIC PLOTTING OF THEORETICAL TIME-DRAWDOWN CURVE AND STRAIGHT-LINE APPROXIMATION

The drawdown in a well with a negligible well loss (Fig. 1(a)) is then:

$$s_w = \frac{Q}{4 \pi T} \left(\log_e \frac{t}{t^*_w} - 0.5772 \right) \ldots\ldots\ldots\ldots (13a)$$

in which $t^*_w = r^2_w \frac{S}{4 T}$, r_w being the effective radius of the well. When the well loss is appreciable (Figs. 1(b) or 1(c)), the drawdown in the well is

$$s_w = \frac{Q}{4 \pi T} \left(\log_e \frac{t}{t^*_w} - 0.5772 \right) + C Q^2 \ldots\ldots\ldots\ldots (13b)$$

Comparing Eq. 13b with Eq. 1, the resistance of the aquifer is

$$B = \frac{\log_e \frac{t}{t^*_w} - 0.5772}{4 \pi T} \ldots\ldots\ldots\ldots (14)$$

According to Eq. 12 or Eq. 13a, S and T may be determined from a series of drawdown observations by plotting values of s against values of the logarithm of t. For sufficiently large values of t, relative to t^* or t^*_w, the points should fall on a straight line. Taking two points on that line to determine the slope,

$$T = \frac{2.30 \, Q \log_{10} \frac{t_2}{t_1}}{4 \pi (s_2 - s_1)} \ldots\ldots\ldots\ldots (15a)$$

[9] "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage," by Charles V. Theis, *Transactions*, Am. Geophysical Union, 1935, Pt. II, pp. 519–524.

[10] "The Flow of Homogeneous Fluids Through Porous Media," by M. Muskat, McGraw-Hill Book Co. Inc., New York, N. Y., 1937, p. 667.

Eq. 15*a* can be simplified further by choosing, arbitrarily the two points one log cycle apart. Then, $\log_{10} \frac{t_2}{t_1} = 1$, and

$$T = \frac{2.30\,Q}{4\,\pi\,(s_2 - s_1)} \qquad (15b)$$

Knowing T, theoretically the value of S may now be determined from the intercept of the straight line with the zero-drawdown line because at this point $(t, 0)$—

$$\log_e \frac{4\,T\,t}{r^2\,S} = 0.5772 \qquad (16)$$

from which

$$S = \frac{4\,T\,t}{r^2\,e^{0.5772}} = \frac{2.25\,T\,t}{r^2} \qquad (17)$$

Application of Theory to a Simple Drawdown Test

Fig. 4 is a semilogarithmic graph of data from a simple drawdown test of a pumping well and a near-by observation well in glacial outwash near Meadville, Pa. The pumping well is of the gravel-wall type and has 15 ft of 18-in. screen between depths of 49 ft and 64 ft. During the test it was pumped at $Q = 1{,}350$ gal per min, or about 3.0 cu ft per sec. Observations of drawdown were made periodically by an air line in the pumping well. An automatic gage gave a continuous record of the drawdown and subsequent recovery in an observation well 1,200 ft away from the pumping well. The data for the drawdown period are plotted as open circles. The data for the recovery period are plotted as solid circles.

The transmissibility may be determined from the slope of either straight line. In both cases the change in drawdown over one log cycle is 2.27 ft. According to Eq. 15*b*, with Q equal to 3.0 cu ft per sec, the value of T is $\frac{2.30 \times 3.0}{4\,\pi \times 2.27} = 0.24$ sq ft per sec. The storage coefficient may be determined from the intercept of the upper straight line with the zero-drawdown line by substituting $t = 693$ sec in Eq. 17. The result of this calculation is $S = \frac{2.25 \times 0.24 \times 693}{1.44 \times 10^6} = 0.00026$.

Assuming the porosity of the sand to be 35% and its effective thickness about 100 ft, the apparent water compressibility is computed as follows: By Eq. 6*a*,

$$\beta' = \frac{S}{\gamma\,n\,b} = \frac{0.00026}{62.4 \text{ (lb per cu ft)} \times 0.35 \times 100 \text{ (ft)}}$$

$$= \frac{1}{8{,}400{,}000 \text{ (lb per cu ft)}} = \frac{1}{58{,}000 \text{ (lb per cu in.)}}$$

[4] The bulk modulus of gas-free water at ordinary temperatures is about 300,000 lb per sq in. The apparent water compressibility (which is the reciprocal of bulk modulus) in this case is then about five times the actual compressibility of water.

Assuming that this value of S holds within the immediate vicinity of the well, the well loss and the factor C may be determined under the further provisional assumption that the effective well radius is equal to the nominal well radius (see Fig. 1(*b*)).

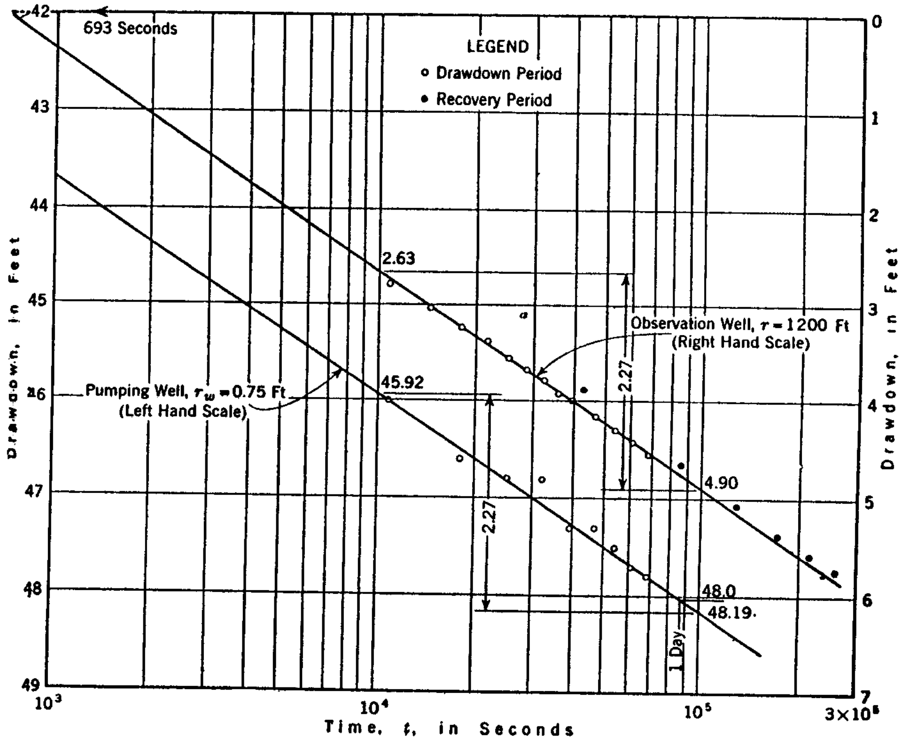


Fig. 4.—Semilogarithmic Plotting of Data from a Simple Drawdown Test of a Pumping Well and a Near-By Observation Well to Determine Transmissibility, Storage Coefficient, and Well Loss

For example, to determine the drawdown for one day, with the use of Fig. 4, solve the formula:

$$s_w = \frac{Q}{4\,\pi\,T}\left(2.303 \log_{10} \frac{4\,T\,t}{r_w^2\,S} - 0.5772\right) + C\,Q^2 \qquad (18)$$

The fraction $\frac{4\,T\,t}{r_w^2\,S} = \frac{4 \times 0.24 \times 86{,}400}{0.5625 \times 2.6 \times 10^{-4}} = 5.67 \times 10^8$; $\log_{10} \frac{4\,T\,t}{r_w^2\,S} = 8.753$; the value in parentheses in Eq. 18 equals $(8.753 \times 2.303) - 0.58 = 19.5$; and $s_w = \frac{3.0 \times 19.58}{4\,\pi \times 0.24} + C\,Q^2 = 19.5 + C\,Q^2 = 48.0$ ft. Finally, $C\,Q^2 = 9.0\,C = 48.0 - 19.5 = 28.5$ ft; from which $C = 3.2 \left(\frac{\text{sec}^2}{\text{ft}^5}\right)$. The foregoing calculations indicate that $B\,Q$ in this case was about 19.5 ft after 24 hours of continuous pumping. The observed drawdown in the pumping well at that time was 48.0 ft, leaving ..5 ft for the well loss. Inasmuch as Q was about 3.0 cu ft per sec, the numerical value of C was approximately 3.2.

Fig. 5 shows how the specific capacity of the well under discussion would vary with the discharge or with time. Because of the relatively high well loss, the one-day specific capacity for $Q = 3$ cu ft per sec is only about 60% of that for $Q = 1$ cu ft per sec. The one-year specific capacity for 3 cu ft per sec

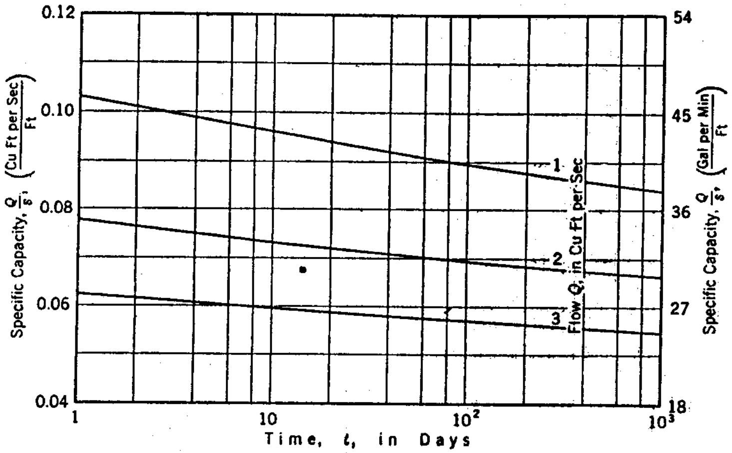


FIG. 5.—VARIATION OF THE SPECIFIC CAPACITY OF PUMPING WELL OF FIG. 4 WITH DISCHARGE AND WITH TIME

is about 65% of that for 1 cu ft per sec. Since at lower discharge rates a greater proportion of the total drawdown is attributable to head loss occurring within the formation (which increases with time while the other component remains constant), the specific capacity shows a greater percentage decline at the lower discharge rates. This fact is shown clearly in Fig. 5. With $Q = 1$ cu ft per sec, the specific capacity declines from about 0.104 cu ft per sec per ft at one day to about 0.086 cu ft per sec per ft at one year—a drop of about 17%. On the other hand, with $Q = 3$ cu ft per sec, the specific capacity declines from about 0.063 cu ft per sec per ft at one day to about 0.056 cu ft per sec per ft at one year—a drop of about 11%.

Fig. 5 illustrates the importance of stating the length of the pumping period during which the discharge remains constant and at the end of which the reported specific capacity is to be determined. It is also important to state the discharge. Too often in the past the specific capacity has been regarded as invariable, only passing attention being given to its variation with discharge and little or nothing being noted about its variation with time. This neglect may perhaps be attributed to the fact that, most commonly measurements of drawdown are made by an air line, with a pressure gage reading to the nearest pound per square inch or an altitude gage reading to the nearest foot no effort being made to interpolate closer than half a scale division. Very often the discharge is allowed to vary unmethodically to obtain several points quickly on the discharge-head curve for checking the characteristics of the pump and motor.

In the example given in this section it was assumed that the gravel envelope was not particularly effective for reducing the drawdown in the vicinity of the well and that therefore the nominal radius of the well screen might be used for the effective radius of the well. Actually, a more practical line of attack is to assume that conditions are as indicated in Fig. 1(c) and then to devise a method of determining the well loss that is independent of the effective well radius, which is to be determined last. The theory of such a method is outlined in the following section.

Theory of Multiple-Step Drawdown Test to Determine Well Loss and Effective Well Radius

Eq. 13b gives the drawdown in an artesian well with an appreciable well loss. It applies to a single drawdown period preceded by a period during which the well is idle. By modifying the second term of the right-hand member, Eq. 13b may be made to apply to increments of drawdown occurring in successive periods, at the beginning of each of which the discharge is increased abruptly.

Fig. 6 depicts the progressive lowering of head in a multiple-step drawdown test. Two sets of drawdown curves are shown: The light lines in Fig. 6(b) show the draw-down that would occur during the successive periods of the test if there were no well loss; and the heavy lines, to which the notations refer, include the well losses, $C\,Q^2$, values of which are indicated in Fig. 6(a).

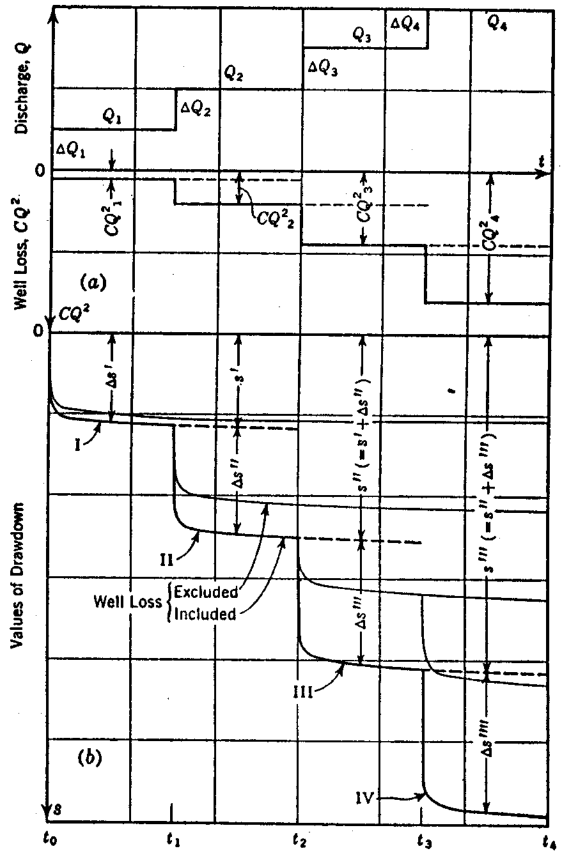


FIG. 6.—VARIATION OF DISCHARGE, DRAWDOWN, AND WELL LOSS IN MULTIPLE-STEP DRAWDOWN TEST

At time $t = t_0$ if the well (which theretofore has been idle) is started pumping at a rate $Q_1 = \Delta Q_1$, the drawdown at any time t thereafter is given by

$$\Delta s' = \frac{\Delta Q_1}{4\pi T}\left(\log_e \frac{t'}{t^*_w} - 0.5772\right) + C\,Q^2_1 \qquad (19a)$$

in which $t' = t - t_0$. If at some later time t_1 the discharge is increased by an amount ΔQ_2 to a new rate Q_2, the second increment of drawdown obeys the relation,

$$\Delta s'' = \frac{\Delta Q_2}{4 \pi T}\left(\log_e \frac{t''}{t^*_w} - 0.5772\right) + C\,(Q^2_2 - Q^2_1) \quad \text{(19b)}$$

with $t'' = t - t_1$.

Similarly, the third increment of drawdown beginning at time t_2 obeys the relation,

$$\Delta s''' = \frac{\Delta Q_3}{4 \pi T}\left(\log_e \frac{t'''}{t^*_w} - 0.5772\right) + C\,(Q^2_3 - Q^2_2) \quad \text{(19c)}$$

in which $t''' = t - t_2$. The generalized equation of this progression is

$$\Delta s^{(i)} = \frac{\Delta Q_i}{4 \pi T}\left(\log_e \frac{t^{(i)}}{t^*_w} - 0.5772\right) + C\,(Q^2_i - Q^2_{i-1}) \quad \text{(20)}$$

in which $t^{(i)} = t - t_{i-1}$.

The value of T might be determined from any one of these equations by plotting $\Delta s^{(i)}$ against $\log_e t^{(i)}$, as before. However, to determine $r^2_w S$, from which to solve for r_w, the constant C must first be found, as this factor shifts the intercept according to the magnitude of Q_{i-1} and Q_i.

Dividing Eqs. 19 and 20 by the respective increments of discharge, and fixing $t' = t'' = t''' \cdots = t^{(i)}$, after simplifying the differences in squares of values of discharge:

$$\left.\begin{aligned} \frac{\Delta s'}{\Delta Q_1} &= B + C\,\Delta Q_1 \\ \frac{\Delta s''}{\Delta Q_2} &= B + C\,(2\,Q_1 + \Delta Q_2) \\ \frac{\Delta s'''}{\Delta Q_3} &= B + C\,(2\,Q_2 + \Delta Q_3) \\ &\cdots \\ \frac{\Delta s^{(i)}}{\Delta Q_i} &= B + C\,(2\,Q_{i-1} + \Delta Q_i) \end{aligned}\right\} \quad \text{(21)}$$

Taking the differences between successive pairs of equations,

$$\left.\begin{aligned} \frac{\Delta s''}{\Delta Q_2} - \frac{\Delta s'}{\Delta Q_1} &= C\,(\Delta Q_1 + \Delta Q_2) \\ \frac{\Delta s'''}{\Delta Q_3} - \frac{\Delta s''}{\Delta Q_2} &= C\,(\Delta Q_2 + \Delta Q_3) \\ &\cdots \\ \frac{\Delta s^{(i)}}{\Delta Q_i} - \frac{\Delta s^{(i-1)}}{\Delta Q_{i-1}} &= C\,(\Delta Q_{i-1} + \Delta Q_i) \end{aligned}\right\} \quad \text{(22)}$$

Considering log t to be variable, Eqs. 21 are the equations of a series of parallel straight lines whose spacings are given by Eqs. 22. Just as the ratio of discharge to drawdown is termed "specific capacity," the ratio of drawdown to discharge may be termed "specific drawdown." Similarly, the ratio of an increment of drawdown to the increment of discharge producing it may be termed "specific incremental drawdown." Eqs. 21, then, give the specific incremental drawdown as a function of time, the factor B increasing with time while the factor C remains constant.

The factor C may be determined from any one of Eqs. 22. Then, knowing C, there may be determined the difference:

$$\frac{\Delta s'}{\Delta Q_1} - \frac{\Delta s^\circ}{\Delta Q_0} = C\,(\Delta Q_0 + \Delta Q_1) \quad \text{(23)}$$

In Eq. 23, $\frac{\Delta s^\circ}{\Delta Q_0}$ is an abbreviation for the limiting value that the specific incremental drawdown approaches as the discharge increment approaches zero and as the well loss consequently becomes negligible. Subtracting Eq. 23 from the first of Eqs. 21, recalling Eq. 14, and keeping in mind that $\Delta Q_0 \rightarrow 0$:

$$\frac{\Delta s^\circ}{\Delta Q_0} = B = \frac{1}{4 \pi T}\left(\log_e \frac{t}{t^*_w} - 0.5772\right) \quad \text{(24)}$$

If the values of specific incremental drawdown given by Eqs. 21 are plotted against log t, as suggested previously a series of straight lines is obtained. Eq. 24 can be plotted on the same graph as a straight line parallel to the others and spaced with reference to the first of the other straight lines in accordance with Eq. 23. Inasmuch as the C-term is lacking in Eq. 24, that equation simply expresses the component of drawdown that occurs in the formation. From its intercept with the zero-drawdown line, then, the effective radius of the pumping well may be determined by the following modification of Eq. 17:

$$r^2_w = 2.25\,\frac{T\,t}{S} \quad \text{(25)}$$

In Eq. 25, S can be determined from observations in a near-by observation well at a known distance r, as shown in Fig. 4.

Data from Multiple-Step Drawdown Test

Figs. 7 and 8 give data from a multiple-step drawdown test run in August, 1943, at Bethpage, Long Island, N. Y. The well that was tested was gravel packed and had 50 ft of 8-in. screen with a No. 60 slot, and with bottom at 350 ft. It was equipped with a 1,200 gal per min deep-well turbine pump. The flow was metered with a propeller-type meter in the discharge line. Water levels inside the casing were measured by an air line with a pressure-gage reading in pounds per square inch. During the first period of the test, measurements of depth to water were made with a weighted steel tape. Thereafter it was not possible to lower the tape to the water surface.

This test had four periods of approximately one hour's duration each. By timing the dial on the nonrecording flow meter with a stop watch, it was possible to secure virtually instantaneous readings of the discharge. Actually, the pumping rate declined slightly during each period of the test as the constantspeed pump adjusted itself to the lowering water level in accordance with the head-discharge characteristic. As these variations were slight, only the average discharge for each period is shown in Fig. 7. Readings of the air-line pressure gage (converted) are indicated by ×'s. The circles plotted one hour after the beginning of each period are interpolated points that are carried over to the discharge-drawdown diagram (Fig. 7(*c*)). Curve A, drawn through these points, is the type of curve ordinarily obtained, drawdown readings being taken only at the end of each period of the test. Customarily, the specific capacity is determined from an average secant of curve A passing through the origin.

Values of drawdown increments taken from Fig. 7(*b*), divided by corresponding increments of discharge, are plotted in Fig. 8 against appropriate values of

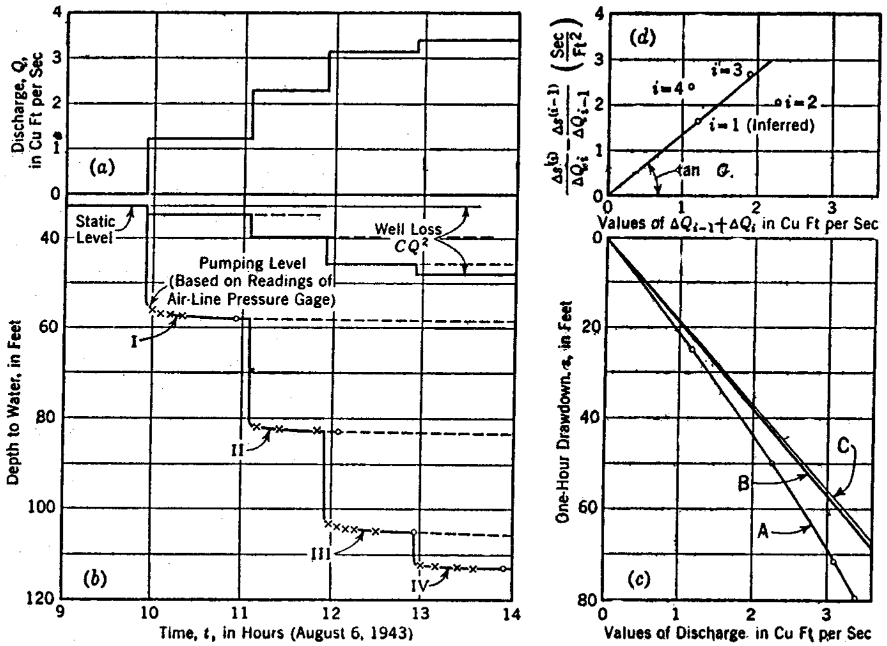
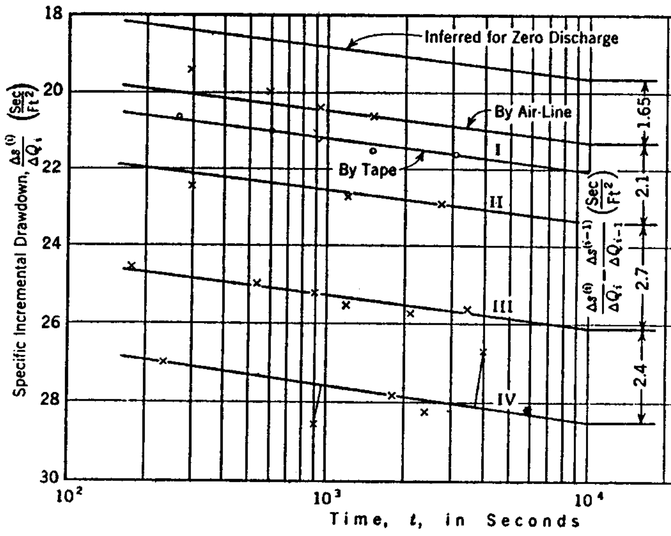


FIG. 7.—DETERMINATION OF WELL LOSS WHEN EFFECTIVE WELL RADIUS IS NOT KNOWN

time on a logarithmic scale. Using the tape readings during the first period as a guide, the slope of the several lines (which theoretically are parallel) is determined. The spacing of the lines in units of feet per cubic feet per second is given in the first column of the tabulation in Fig. 8. Also tabulated are the numbers of the periods, the discharge during each period the increment of discharge at the beginning of each period, and the sums of neighboring increments of discharge. In accordance with Eqs. 22. data in the first and last columns of this tabulation are plotted in Fig. 7(*d*) to determine C. The three experimental points, for i = 2, 3, and 4, show considerable scattering partly



i	Q_i	ΔQ_i	$\Delta Q_{i-1}+\Delta Q_i$
0	0	0	0
1	1.21	1.21	1.21
2	2.28	1.07	2.28
3	3.12	0.84	1.91
4	3.39	0.27	1.11

FIG. 8.—SEMILOGARITHMIC GRAPH OF DATA FROM FIG. 7(*b*)

perhaps because of the insufficiency of the theory but more likely because of inaccuracies of the air-line pressure-gage readings. From the slope of the straight line drawn through the center of mass of the three points, C is found to be 1.35 $\left(\frac{sec^2}{ft^5}\right)$

Knowing C, the amount of well loss during each period of the test may now be determined. For the fourth period of the test, when the discharge was 3.39 cu ft per sec (1,520 gal per min), the well loss is computed as 15.5 ft. During the earlier periods of the test, when the discharge was smaller, the well loss was correspondingly smaller, as indicated in Fig. 7(*a*). Subtracting the appropriate value of well loss from each point plotted in Fig. 7(*c*), curve B is obtained. This curve then represents the discharge-drawdown relation for zero well loss. The straight line C (Fig. 7(*c*)) represents the relation between discharge and drawdown that would be obtained with zero well loss if separate one-hour tests were run at each rate, starting from rest with long intermediate periods of shutdown.

Theoretically, it should be possible to determine the transmissibility of the bed from the slope of the straight lines in Fig. 8; but the situation is complicated somewhat by the lenticular structure of the material penetrated by the well. Although the sand within a few hundred feet of the well is definitely confined, at greater distances it is effectively although somewhat circuitously interconnected with overlying beds of sand. If the setup were more nearly ideal and if there were a near-by observation well from which to determine the value of S, the effective radius of the pumping well might be determined.

Summary

Because of limitations of the data from the two examples given in the paper, a composite of both examples is needed to illustrate the theory of the multiple-step drawdown test completely. With this in mind, the practical application of the theory can be summarized somewhat as follows.

(1) The test should be run following a period in which the well has been inactive, beginning at a fraction of the capacity of the pump and increasing the discharge in steps, each of which is a fair fraction of the pump capacity (Fig. 7). During each period of the test the metered discharge should remain essentially constant. (Small variations in discharge arising from the automatic adjustment of the pump to the declining water level when pumping against a constant discharge pressure are permissible.)

(2) During each period of the drawdown test, frequent readings of the drawdown should be made by air line or, if possible, preferably by a steel tape or by an electric-contact device. If an air line is used, care should be taken to use a reliable pressure gage that has been calibrated and to read it to the nearest fifth or tenth of a scale division.

(3) Frequent drawdown readings should also be taken in one or more observation wells tapping the same sand. If the screen of the pumping well does not completely penetrate the sand, the nearest observation well for this purpose should not be closer than about twice the sand thickness from the pumping well.

(4) Plot the data obtained under item (3) on a semilogarithmic graph (Fig. 4), using increments of drawdown against logarithm of time Determine the transmissibility from the slope of the straight lines and the storage coefficient from their average intercept.

(5) Plot the data obtained under items (1) and (2) on rectangular coordinates (Figs. 7(*a*) and 7(*b*)). Extrapolate the drawdown curve for each period through the following period to determine the increments of drawdown.

(6) Plot values of specific incremental drawdown against the logarithm of time on semilogarithmic paper (Fig. 8). Draw parallel straight lines through the plotted points. (Extensions of these straight lines should check the extrapolations on the other graph. Secondary adjustments may be made to improve the extrapolations.)

(7) Plot differences of specific incremental drawdown given by neighboring lines against the sum of neighboring discharge increments (Fig. 7(*d*)). Determine C from the slope of the straight line through the origin and through the center of mass of the plotted data.

(8) Compute the well loss for each period of the test from CQ^2.

(9) Infer the limiting straight-line relation for zero discharge (Fig. 8) and from its intercept with the zero-drawdown line, using the value of storage coefficient determined under item (4), compute the effective radius of the well.

If the storage coefficient and transmissibility of the bed and the effective radius of the well are determined, it is possible to compute the resistance, B, at any time. Knowing the factor C, it is possible to compute the well loss. Combining the two, the total drawdown in the pumping well may be determined for any time and for any pumping rate.

In extensive artesian aquifers such predictions of drawdown are often trustworthy for periods of several months or a few years, but longer-term predictions must be based upon further consideration and closer evaluation of the outer "boundary conditions" of the aquifer. In local artesian beds this type of analysis may be required even for short-term predictions. In either case the concepts and procedures advanced in this paper should constitute useful implements, although not displacing in any way that knowledge of the geology and hydrology of an aquifer that is so necessary for a complete understanding of its performance.

The procedure itemized in this "Summary" should make possible, at any time during the life of a well, the accurate determination of both components of its specific drawdown, thus, for example, facilitating the recognition of the effects of encrustation of the screen or sand packing of the gravel wall, which too often have been ascribed to "depletion of the sand." Furthermore, it should enable the evaluation of the effectiveness of gravel packing and of the various development operations practiced in well construction. Through the accumulation of data, as wells are developed and placed in operation, this procedure should greatly aid in the selection of the proper gravel size and the appropriate screen opening so that the efficiency of wells will be increased and much needless waste of pumping energy will be prevented. Similarly, where the gravel wall is not called for, unnecessary expenditures for this type of construction may be avoided by referring to cases that have been tested under similar circumstances. Through predictions of the trend of pumping levels with time, proper selection of the pump and motor may be made that will give optimum performance throughout the life of the well, thus avoiding the wasteful practice of operating a pump with its discharge throttled to keep within the limits set by the diminishing capacity of the well.

The decline in production of oil wells is perhaps even more troublesome than that in the production of water wells. Often, it is difficult to determine whether the decline is due merely to depletion of the reservoir or whether it is due to the plugging of the perforations in the "liner," to the transportation of the fines, to the deposition of asphaltic substances, or to other causes. With some modifications the procedure outlined in this paper can be applied to oil wells as an aid in answering such questions. However, more accurate determinations of fluid level would be required than are now generally feasible while pumping steadily at different rates of production.

Acknowledgment

The writer wishes to express his appreciation of their helpfulness to O. E. Meinzer, chief of the Ground-Water Division, Water Resources Branch, United States Geological Survey and to M. L. Brashears, Jr., district geologist, in charge of ground-water investigations in New York and New England, under whose direction he worked during the development of this paper.

APPENDIX. NOTATION

The following letter symbols conform essentially with American Standard Letter Symbols for Hydraulics (ASA—Z10.2—1942) and with ASCE *Manual of Engineering Practice No. 22* on "Soil Mechanics Nomenclature"

B = "hydraulic resistance" of formation, head loss per unit discharge;
b = thickness of confined sand bed;
C = coefficient in term $C\,Q^2$ expressing "well loss," a component of drawdown, the other term of which is $B\,Q$;
k = transmission constant, or "coefficient of permeability";
n = porosity of sand;
Q = discharge of well;
ΔQ_i = increment of discharge; $i = 1, 2, 3$,
r = radial distance from axis of well;
r_w = "effective radius" of well;
S = "coefficient of storage" = $(b/V)(dV_w/ds)$;
s = drawdown at distance r, the difference between initial head and head at time t at that distance;
s_w = drawdown at r_w, according to theoretical logarithmic distribution;
$\Delta s^{(i)}$ = increment of drawdown produced by ΔQ
$\Delta s^{(i)}/\Delta Q_i$ = "specific incremental drawdown" during i^{th} period of test;
$\Delta s^{\circ}/\Delta Q_0$ = limiting value of specific incremental drawdown for discharge approaching zero (= B);
T = "transmissibility" of sand bed = $k\,b$;
t = time;
t^* = "inflectional time" = $r^2\,S/4\,T$;
u = $r^2\,S/4\,T\,t = t^*/t$, a nondimensional variable;
V = volume;
V_w = volume of water;
$W(u)$ = "well function" of u, or the negative "exponential integral" of $-u$, for which tables are available;
α = "compressibility" of solid skeleton of sand bed, relative decrease in thickness per unit increase of vertical component of compressive stress in sand bed;
β = compressibility of water in sand bed;
β' = apparent compressibility of water = $\beta + \alpha/n$; and
γ = specific weight of water.

DISCUSSION

N. S. BOULTON,[11] ESQ.—The importance of carefully recording both the small variations in pumping level, which may occur during pumping tests at constant discharge, and the duration of the test, are appropriately stressed in this paper. From such information it is possible to predict, as the author has shown, the probable steady decline in specific capacity "for periods of several months or a few years" when the well is pumped at constant discharge. It is important to remember, however, that the accuracy of this prediction depends essentially on the assumption that the compressibility of the aquifer (which enters into the coefficient of storage) has the same value for the very small pressure releases which occur at large distances from the pumped well as for the comparatively large pressure releases near to the well. It would be appreciated if the author could present evidence in support of this assumption, based on long-period observations of declining well levels. In addition, it would be interesting to know whether the author has been able to check the values for "well loss" by direct estimates of the pipe friction loss as the water flows inside the well casing and also of the loss of head due to the screen.

For the fourth period of the test at Bethpage, Long Island, N. Y., the depth of water in the well was apparently about 238 ft. Allowing for the water entering the well uniformly along the bottom 50 ft, a reasonable estimate (from a usual formula) for the head lost in pipe friction in the 8-in.-diameter tube is about 10.5 ft, including 1.5 ft for the velocity head. The computed well loss (see heading, "Data from Multiple-Step Drawdown Test") is stated to be 15.5 ft, which leaves 5 ft for the loss due to the screen. It is easy to calculate the latter loss on the assumption of flow through a uniform permeable medium outside the screen to which Darcy's law may be applied. Thus, for long vertical slots spaced equally around the circumference of the well, it can be shown from the potential solution for the flow net that the head loss due to the restricted inlet area provided by a slotted tube is closely given by:

$$h = \frac{Q}{2\,\pi\,N\,b\,k} \log_e \left(\frac{2}{1 - \cos \nu\,\pi}\right) \quad (26)$$

in which N is the number of vertical slots around the circumference of the tube; and ν is the slot-width ratio or width of slot divided by the distance between the centers of two adjacent slots.

According to Eq. 26, the head loss is proportional to the discharge and, for a given slot-width ratio, inversely proportional to the number of slots.

If Q = 3.39 cu ft per sec and b = 50 ft, as in the fourth period of the Bethpage test, and if k = 0.004 ft per sec (as deduced from Fig. 8), assuming $\nu = \frac{1}{4}$ (since the dimensions of the slotted tubing are not given in the paper), it is found on substitution in Eq. 26 that $h = 5.2/N$ ft. For one hundred slots, each 0.063 in. wide, uniformly spaced around the circumference, h = 0.052 ft which is negligible. On the other hand, if the slots are arranged in batteries

[11] Lecturer in Chg., Dept. of Civ. Eng., Univ. of Sheffield, Sheffield, England.

numbering, say, ten in the circumference, the batteries being 0.5 in. wide with 2 in. between them, $h = 0.52$ ft which is still small.

It should be emphasized that this calculation makes no allowance for any clogging of the slots. Such clogging may account for the discrepancy between the small calculated screen loss and the value of 5 ft deduced from the test result.

CARL ROHWER,[12] M. ASCE.—The serious depletion of ground-water supplies in many areas during World War II has focused attention on the problems of ground-water hydrology. In this connection the investigations of the engineers of the Water Resources Branch of the U. S. Geological Survey are adding important information regarding the characteristics of wells and the capacity of ground-water formations. The analysis of drawdown tests of artesian wells by the author is a valuable contribution to this subject.

The writer is in agreement with the objectives of the author's investigations but he is of the opinion that the analysis of the problem would have been simplified if some of the factors that have only a slight effect on the results had been ignored. Under most conditions met with in the field of engineering the compressibility of water can be ignored. The coefficient is approximately 4×10^{-6} per pound pressure at ordinary temperatures and pressures. A reduction in pressure of 10 lb per sq in. would increase the volume of 1 cu ft of water by only 0.00004 cu ft, a difference of 1 in 25,000. In view of the large unavoidable errors involved in other measurements it seems that this factor could well be neglected. The same may be stated of the compression of the aquifer. As indicated by the author (see heading, "Application of Theory to a Simple Drawdown Test"), the combined effect of compressibility of the water-bearing formation is only five times the actual compressibility of water. Consequently, the combination of these two factors would produce a change of only 1 in 5,000 for a drop in pressure of 10 lb (approximately 23 ft). If the change in pressure were increased to 100 ft the effect produced by the compressibility of the water and aquifer would not be significant.

In reference to the tests on a shallow well at Meadville, Pa., the author states in the sentences following Eq. 18, that:

> "The foregoing calculations indicate that $B\,Q$ in this case was about 19.5 ft, after 24 hours of continuous pumping. The observed drawdown in the pumping well at that time was 48.0 ft, leaving 28.5 ft for the well loss."

Such a large well loss seems unusual for an inflow of 1,350 gal per min through 15 ft of 18-in. screen unless the screen were badly encrusted or improperly perforated. Immediate steps should be taken to improve the performance of the screen in this well.

In the solution of problems involving many variables of which only a few can be determined by direct measurement, the use of multiple equations provides a method of determining the unknowns. However, there are difficulties inherent in this method which may lead to contradictory or inconsistent results.

[12] Senior Irrig. Engr., Div. of Irrig., Soil Conservation Service, U. S. Dept. of Agriculture, Fort Collins, Colo.

As shown by the author in reference to the determination of C (see heading, "Data from Multiple-Step Drawdown Test"), there is considerable scattering in the values obtained for the "specific incremental drawdown," Fig. 7(d). No doubt, this is due in part to the inaccuracies in the drawdown readings. If this method were used on problems in which all readings could be made accurately, consistent results should be expected. Since this is not generally true, the multiple-equation method results in solutions in which the final answers may have errors greatly in excess of the observed data. The writer is not aware of the mathematical basis for this assumption, but he observed the same effect when attempting to use a similar method to determine the values of the factors involved in the seepage from canals. The conclusion was reached that, in the elimination of variables from the series of equations by subtraction, the variables eliminated were forced to conform exactly to the law; and, as a result, all the discrepancies accumulated and finally appeared in the solution of the unknown. A solution based on another pair of equations may, therefore, yield a result widely different from the first one.

Since the author has had the opportunity to observe how the solutions vary when he uses different equations it would be of interest to study the mathematical principles causing the variations. No doubt rules could be formulated which would make it possible to obtain more consistent results from the observed data. Such an analysis would be useful in the solution of problems in other fields of engineering.

R. M. LEGGETTE,[13] AFFILIATE, ASCE—Although it covers a highly technical subject, this paper clearly demonstrates the practical importance of a number of factors of well design. It seems desirable to emphasize these practical considerations because they are often given too little attention. Frequently water works men and well-drilling contractors greatly belittle or fail to recognize the magnitude of what Mr. Jacob calls "well loss."

It is obvious, of course, that the water level in a pumping well must be lower than the water level immediately outside the well. In many wells, much of this difference in head is screen friction loss which results from the use of a poorly designed screen. This difference in head is sometimes presumed to be only a few inches, or a fraction of a foot; however, actual observations have shown that in some wells the well loss is a considerable part of the total drawdown. Thus, from the point of view of economy of operation, well loss may be an important factor.

The paper also indicates the desirability of increasing the effective radius of a "sand and gravel" well by development to remove the fine material surrounding the screen, or by artificial gravel packing. It should be noted that the advantages of development or gravel packing may be largely overcome if an inefficient well screen is used.

The process of development by surging, swabbing, and brushing is being used more and more in uncased wells (rock wells), the walls of which apparently become "mudded up" during the drilling process. This clogging of the uncased wall of the well has the same effect as an inefficient well screen in a "sand

[13] Cons. Ground-Water Geologist, New York, N. Y.

and gravel" well. The well loss in many wells of this type has been greatly reduced by development.

From the point of view of economy of pumping well water, this paper indicates the following: A well should be of large diameter; it should be extensively developed; and, if a well screen is used, it should be designed so as to produce a minimum of screen friction loss.

M. R. Lewis,[14] M. ASCE.—A valuable method has been presented, in this paper, for analyzing the capacity of artesian wells in spite of the necessary assumptions that there is uniformity in the aquifer and that the total supply to the well is drawn from the aquifer by the release of elastic forces. Such theoretical or mathematical treatments of the flow of fluids assist greatly in the understanding of practical problems even though the latter seldom are based on ideal conditions.

The multiple-step test proposed by the author should give very useful information on the points mentioned in the "Summary" wherever the assumptions are approximately fulfilled. It is hoped that the author will explore the possibility of a similar analysis under other conditions. Two important types of wells that might be studied are those of a simple water-table situation and those in which the bed overlying an aquifer is relatively impermeable but permits recharge from the soil surface surrounding the well.

It appears to the writer that two other factors besides the compaction of the aquifer are important elements in making the "apparent compressibility, β'," greater than the compressibility of water, β. These are (a) the increase in volume of the solid material of the aquifer because of the reduced hydrostatic pressure and (b) the reduction in the pore space because of the deformation of the solid particles by reason of the increased pressure on the mineral skeleton. In his earlier paper,[8] the author mentioned these factors but, apparently, considered them to be of negligible importance. Whether they are, or are not, important makes no difference in the author's analysis.

C. E. Jacob,[15] Assoc. M. ASCE.—Although few in number and brief, the discussions have added much to the paper and, moreover, have suggested the direction that further work might profitably take. The writer wishes to thank those who have contributed.

The question raised by Mr. Boulton is a pertinent one—regarding the need for evidence to support the assumption that the compressibility of the aquifer has the same value near to, and far from, the pumped well despite the wide range of pressure release. The writer knows of no close observations of long-period decline under constant and continuous pumping that might clarify this problem. Of course, it is to be expected that an unconsolidated sand would have a variable compressibility, depending on the rate and magnitude of the loading occasioned by the release of pressure. Moreover sight should not be lost of the fact that the flexural rigidity of the overlying beds complicates the problem, especially in the immediate vicinity of pumped wells tapping deep aquifers. Fortunately, however, as long as the compressibility (thus modified)

[14] Irrig. Engr., Bureau of Reclamation, Boise, Idaho.

[15] Senior Hydr. Engr., Water Resources Branch, U. S. Geological Survey Washington, D. C.

may be assumed reasonably constant in time at a given distance the uncertainties arising from ignorance of its absolute value are reflected only in the degree of approximation of r_w. Actually, in predictions of future drawdown based on the theory of an elastic aquifer, the product $S\,r^2_w$ is used. This product is determined empirically and it is not necessary to break it down into S and r_w, except to compare different wells. If S varies with t, for one reason or another, then some other theory must be used or the elastic theory must be modified.

Mr. Boulton's estimate of 10.5 ft for the friction and velocity head losses accompanying the upward flow inside the well casing of the Bethpage well narrows down the well loss to that part that may be termed "screen loss." His calculation of the "convergence loss" under laminar flow into an assumed system of vertical slots shows that such loss would be quite small. Actually the openings in the screen are in the form of a helix, but the "convergence loss" there should be of the same or even of a smaller order of magnitude. Clogging of the slots is a distinct possibility, as pointed out by Mr. Boulton. Furthermore, departure from laminar flow may begin as the water passes through the screen openings, especially if they are clogged.

By calculation Mr. Rohwer shows that the combined relative compression of the aquifer near Meadville, Pa., and its contained water is only about 1 in 5,000 for a 23-ft drop in head. He states:

> "In view of the large unavoidable errors involved in other measurements it seems that this factor [compressibility of water] could well be neglected. The same may be stated of the compression of the aquifer."

Perhaps Mr. Rohwer has in mind estimating the ultimate or steady-state discharge of wells in a limited aquifer, in which case the volume of water derived from artesian storage might soon become small by comparison with the volume drawn through the aquifer from an outer boundary. However, until that steady state of flow is established, water is withdrawn from storage through its elastic expansion and through the concomitant compression or compaction of the aquifer. Indeed, in extensive and deep-lying aquifers such as the Dakota sandstone, the supply may be furnished entirely from storage for decades. Because of the tremendous volume of water in such an aquifer and because of the sizable lowering of head that may be produced, the total volume of water withdrawn from storage through wells may itself reach an astounding magnitude. In any event, however insignificant this factor may appear, the study of the transient behavior of an elastic aquifer, on which the paper is based, requires an appraisal of its magnitude.

With reference to the large well loss of 28.5 ft at 1,350 gal per min in the well near Meadville, Pa., the suggestion is offered by Mr. Rohwer that the screen may be encrusted or improperly perforated. As the well was newly constructed when tested, it would seem that sand packing of the gravel envelope and of the screen might be the explanation. The formation is a uniform fine sand—difficult to handle in drilling and developing a well.

Mr. Rohwer adds a valuable point in remarking on the unavoidable magnification of errors through elimination of variables by subtraction from the series of equations. He justifiably emphasizes the need for care in treating such data as those presented in the paper. Whereas from a theoretical standpoint, with ideal data, the procedure outlined in the paper is sound; in practice it needs modification. Higher precision of measurement may warrant the assumption that the drawdown obeys the law $s_w = B\,Q + C\,Q^n$, introducing a third unknown, the exponent n (<2), to be determined by trial-and-error computation, or by graphical procedure together with the coefficients B and C.

An important point is raised by Mr. Leggette—that the advantages of gravel packing or developing a well may be offset by poor design or improper choice of screen. The writer feels that amassing empirical values of C and of r_w, together with pertinent data on the details of design and construction of wells, may eventually make possible the accurate appraisal of these various factors. The selection of screen type, slot opening, and gravel size—and even the determination of whether or not a gravel envelope is required—may be lifted from the realm of guesswork to a rational plane through the future study of existing and newly constructed wells and through the measurement of their characteristics of performance, due consideration being given to the transient behavior of the aquifer.

In emphasizing the magnitude of well losses, condemnation of the well driller is not intended, for much of the friction loss in and near a well is unavoidable and will never be entirely eliminated. Nevertheless, it behooves the engineer and the well-drilling contractor alike to strive for as efficient design and construction as possible to meet the stringencies of economic demands. The points summarized by Mr. Leggette thus are objectives toward which progress should be made.

Mr. Lewis suggests that similar analyses be made for unconfined flow under simple water-table conditions, and for confined flow in which recharge from the soil surface occurs through a relatively impermeable confining bed. To the writer's knowledge a satisfactory analysis of nonsteady unconfined flow has not been given. Even in the case where the storage coefficient (S, ultimately approaching "specific yield") is constant, there are insuperable difficulties. Only by analogy to confined flow, and then in cases where the maximum drawdown is but a small fraction of the initial depth of flow, has an approximate solution been obtained. It may be stated, however, that even in the absence of well losses the specific capacity of a water-table well would vary with the discharge. the curve of drawdown versus discharge at constant time being a parabola under certain approximative assumptions. Further work should be done on this problem, both in the laboratory and in the field.

A solution has been given for the nonsteady radial flow toward a steadily discharging well in a leaky confined aquifer.[16] The leakage is assumed proportional to the drawdown. Whether this is exactly the condition Mr. Lewis has in mind is not known, but in the early phase of a transient state such a system acts like an ideal elastic aquifer without leakage Accordingly, a short multiple-step drawdown test could be analyzed under those conditions on the basis of the elastic theory, although long-term predictions would consider the leakage.

[16] "Radial Flow in a Leaky Artesian Aquifer," by C. E. Jacob, *Transactions*, Am. Geophysical Union, Vol. 27, 1946, Pt. II, pp. 198–205.

CALCULATION OF C

JACOB (1946)

1. STEP 1 : $t_0 < t < t_1$, $Q = Q_1$, $\Delta Q_1 = Q_1$

$$s_w = \Delta s_1 = \frac{\Delta Q_1}{4\pi T}\left(\ln\left\{ \frac{4T(t-t_0)}{r^2 S} \right\} - 0.5772 \right) + C Q_1^2 \qquad \text{—(19A)}$$

STEP 2 : $t_1 < t < t_2$, $Q = Q_2$, $\Delta Q_2 = Q_2 - Q_1$

$$s_w = \frac{\Delta Q_1}{4\pi T}\left(\ln\left\{ \frac{4T(t_1-t_0)}{r^2 S} \right\} - 0.5772 \right)$$

$$+ \frac{\Delta Q_2}{4\pi T}\left(\ln\left\{ \frac{4T(t-t_1)}{r^2 S} \right\} - 0.5772 \right) + C Q_2^2$$

and $\Delta s_2 = s_w - \Delta s_1(t_1)$

$$= \frac{\Delta Q_2}{4\pi T}\left(\ln\left\{ \frac{4T(t-t_1)}{r^2 S} \right\} - 0.5772 \right) + C\left(Q_2^2 - Q_1^2\right) \qquad \text{—(19B)}$$

STEP 3 : $t_2 < t < t_3$, $Q = Q_3$, $\Delta Q_3 = Q_3 - Q_2$

$$s_w = \frac{\Delta Q_1}{4\pi T}\left(\ln\left\{ \frac{4T(t_1-t_0)}{r^2 S} \right\} - 0.5772 \right)$$

$$+ \frac{\Delta Q_2}{4\pi T}\left(\ln\left\{ \frac{4T(t_2-t_1)}{r^2 S} \right\} - 0.5772 \right)$$

$$+ \frac{\Delta Q_3}{4\pi T}\left(\ln\left\{ \frac{4T(t-t_2)}{r^2 S} \right\} - 0.5772 \right) + C Q_3^2$$

and $\Delta s_3 = s_w - s_w(t_2)$

$$= \frac{\Delta Q_3}{4\pi T}\left(\ln\left\{\frac{4T(t-t_2)}{r^2S}\right\} - 0.5772\right) + C\left(Q_3^2 - Q_2^2\right) \qquad \text{—(19C)}$$

The generalization is :

$$\Delta s_{(i)} = s_w - s_w\left(t_{(i-1)}\right)$$

$$= \frac{\Delta Q_{(i)}}{4\pi T}\left(\ln\left\{\frac{4T(t-t_{(i-1)})}{r^2S}\right\} - 0.5772\right) + C\left(Q_{(i)}^2 - Q_{(i-1)}^2\right)$$

—(20)

Note : Jacob refers to $(t - t_{(i-1)})$ as $t^{(i)}$

2. Divide through by the respective increments of discharge :

STEP 1 : $$\frac{\Delta s_1}{\Delta Q_1} = \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t-t_0)}{r^2S}\right\} - 0.5772\right) + C Q_1 \qquad \text{—(21A)}$$

STEP 2 : $$\frac{\Delta s_2}{\Delta Q_2} = \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t-t_1)}{r^2S}\right\} - 0.5772\right) + C\,\frac{(Q_2^2 - Q_1^2)}{\Delta Q_2}$$

Now $Q_2 = Q_1 + \Delta Q_2$

$$\therefore \quad \frac{\Delta s_2}{\Delta Q_2} = \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t-t_1)}{r^2S}\right\} - 0.5772\right) + C\left(2Q_1 + \Delta Q_2\right)$$

—(21B)

STEP 3 :
$$\frac{\Delta s_3}{\Delta Q_3} = \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t-t_2)}{r^2 S}\right\} - 0.5772\right) + C\,\frac{(Q_3^2 - Q_2^2)}{\Delta Q_3}$$

Now $Q_3 = Q_2 + \Delta Q_3$

$$\therefore \quad \frac{\Delta s_3}{\Delta Q_3} = \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t-t_2)}{r^2 S}\right\} - 0.5772\right) + C\left(2Q_2 + \Delta Q_3\right) \qquad \text{—(21C)}$$

The generalization is :

$$\frac{\Delta s_{(i)}}{\Delta Q_{(i)}} = \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t-t_{(i-1)})}{r^2 S}\right\} - 0.5772\right) + C\left(2Q_{(i-1)} + \Delta Q_{(i)}\right)$$

└ Jacob calls this B ┘ —(2D)

3. Take the difference between each successive pair, at the ends of the steps

STEP 2 − STEP 1 :

$$\frac{\Delta s_2}{\Delta Q_2} - \frac{\Delta s_1}{\Delta Q_1} = \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t_2-t_1)}{r^2 S}\right\} - 0.5772\right) + C(2Q_1 + \Delta Q_2)$$

$$- \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t_1-t_0)}{r^2 S}\right\} - 0.5772\right) - CQ_1$$

$$= \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t_2-t_1)}{r^2 S}\right\} - \ln\left\{\frac{4T(t_1-t_0)}{r^2 S}\right\}\right)$$

$$+ C\left(Q_1 + \Delta Q_2\right) \qquad \text{—(22A)}$$