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ESTIMATING THE TRANSMISSIBILITY OF AQUIFERS FROM THE SPECIFIC CAPACITY OF WELLS

By Charles V. Theis, Russell H. Brown, and Rex R. Meyer

ABSTRACT

The specific capacity of a well can be used as a basis for estimating the coefficient of transmissibility of the aquifer tapped by the well. From assumed values for the hydrologic constants of the aquifer, separate formulas including a term for specific capacity are developed for the transmissibility of water-table and artesian aquifers. From a chart relating the well diameter, the specific capacity of the well, and the coefficients of transmissibility and storage, the transmissibility of the aquifer can be estimated from the known specific capacity of the well or the specific capacity of the well can be estimated from the known transmissibility of the aquifer. These methods are subject to limitations but are useful means of approximation.

THE GENERAL RELATIONSHIP BETWEEN TRANSMISSIBILITY AND SPECIFIC CAPACITY

In many ground-water investigations, especially those of a reconnaissance type, the specific capacities of wells provide the only basis for estimating the transmissibility of the aquifers tapped by the wells. Generally speaking, high specific capacities indicate an aquifer having a high coefficient of transmissibility, T, and low specific capacities indicate an aquifer having a low T. However, a precise correlation between the specific capacities of wells and the T values of the aquifers they tap has not yet been established.

The specific capacity of a well cannot be an exact criterion of T in the vicinity of the well because, obviously, the yield of the well per foot of drawdown is also a function of other factors such as the diameter of the well, the depth to which the well extends into the aquifer, the type and amount of perforation in the well casing, and the extent to which the well has been developed. However, estimates of T that are based on the specific capacities of wells should be reasonably reliable and could be made without the elaborate tests necessary for precise determinations. Therefore, if developed within the limits of idealized assumptions, a formula expressing the theoretically exact relationship between the specific capacity of a well and the transmissibility of the aquifer which the well taps would be highly useful in the making of reconnaissance ground-water studies provided the theoretical formula is empirically modified for prevailing field conditions.

ESTIMATING THE TRANSMISSIBILITY OF A WATER-TABLE AQUIFER FROM THE SPECIFIC CAPACITY OF A WELL

By Charles V. Theis

The relation between the discharge of a well and the water-level drawdown a short distance from the well is given by an equation derived by Theis (1935). The value of u in that equation is small provided r is small, T and S are within the range of values for fairly productive aquifers, and t is at least several hours. For the purpose of this paper, the Theis formula can be written with negligible error as follows:

$$T=\frac{114.6Q}{s}\left[-0.577-\log_e\left(\frac{1.87r^2S}{Tt}\right)\right]. \qquad (1)$$

The computation can be made somewhat simpler by substituting values for S and T that are within the range of fairly productive values for water-table aquifers. However, if corrections for these values are included, the formula remains general. Thus, if $T=1{,}000$ gpd per ft, $S=0.2$, and $t=1$ day, the formula for an average water-table aquifer corrected for variations of that aquifer from average is

$$T=\frac{114.6Q}{s}\left[-0.577-\log_e\left(\frac{1.87r^2\cdot 0.2}{100{,}000}\cdot\frac{S\cdot 100{,}000}{0.2\cdot T}\cdot\frac{1}{t}\right)\right]$$

$$=\frac{114.6Q}{s}\left[-0.557-\log_e\left(\frac{(3.74r^2\cdot 10^{-6})(5S)}{(T\cdot 10^{-5})t}\right)\right]$$

$$=-\frac{66Q}{s}+\frac{264Q}{s}\Big[-\log_{10}(3.74r^2\cdot 10^{-6})-\log_{10}5S$$

$$+\log_{10}(T\cdot 10^{-5})+\log_{10}t\Big].$$

Therefore,

$$T-\frac{264Q}{s}\log_{10}(T\cdot 10^{-5})=-\frac{66Q}{s}+\frac{264Q}{s}$$

$$[-\log_{10}(3.74r^2\cdot 10^{-6})-\log_{10}5S+\log_{10}t].$$

Let

$$T'=T-\frac{264Q}{s}\log_{10}(T\cdot 10^{-5}) \qquad (2)$$

then

$$T'=-\frac{66Q}{s}+\frac{264Q}{s}[-\log_{10}(3.74r^2\cdot 10^{-6})-\log_{10}5S+\log_{10}t]$$

$$=\frac{Q}{s}[-66-264\log_{10}(3.74r^2\cdot 10^{-6})-264\log_{10}5S+264\log_{10}t].$$

Let

$$K=-66-264\log_{10}(3.74r^2\cdot 10^{-6}) \qquad (3)$$

then

$$T'=\frac{Q}{s}(K-264\log_{10}5S+264\log_{10}t). \qquad (4)$$

Values of K, computed for selected values of r, are as follows:

r (ft)	K	r (ft)	K
0.25	1,684	20	680
.50	1,524	30	588
1.0	1,367	40	521
5.0	996	50	469
10	838		

The foregoing formulas indicate the importance of both the storage coefficient and the duration of pumping when the coefficient of transmissibility is estimated from a single measurement of drawdown in an observation well. If $S=0.2$, the influence of the S term is zero because the formula was derived on that basis. However, if $S=0.1$, the S term would equal $-264 \log_{10} 5S=-264 \log_{10} 0.5=80$, or about 8 percent of the constant, K, for $r=5$ feet, and if $S=0.3$, the S term would equal -45, or about -4.5 percent of the same value for K. Provided S is known, the correction can be made, but if S is unknown, the error for a water-table aquifer (for which S ranges from 0.1 to 0.3) probably will be smaller than the errors inherent in the method. Although the correction for the duration of pumping also is comparatively small, it presumably should be made if, as in many cases, the duration is known. For an artesian aquifer, S is very small and the S term correction will be large, making it inadvisable to apply the formula (in its present form) for artesian conditions; for if $S=0.001$, the S term would be about double K for $r=5$ feet.

The coefficient of transmissibility cannot be determined explicitly from the computed values of T'. However, from charts giving the values of T' for various values of T and Q/s, the value of T can be ascertained from known values of T' and Q/s. Such a chart is shown in figure 99.

Thus, within the limits of the idealized assumptions, the coefficient of transmissibility of a water-table aquifer apparently can be computed without great error from a single measurement of drawdown in an observation well that is a short distance from a pumped well, even if the coefficient of storage is not known. However, the information generally available concerns the specific capacity of the pumped well. In the foregoing formulas Q represents the discharge of the pumped well and s is the drawdown in a nearby observation well at a distance r from the pumped well. Obviously, the drawdown in the pumped well bears a relationship to the drawdown a short distance from the well. If this relationship can be ascertained approximately,

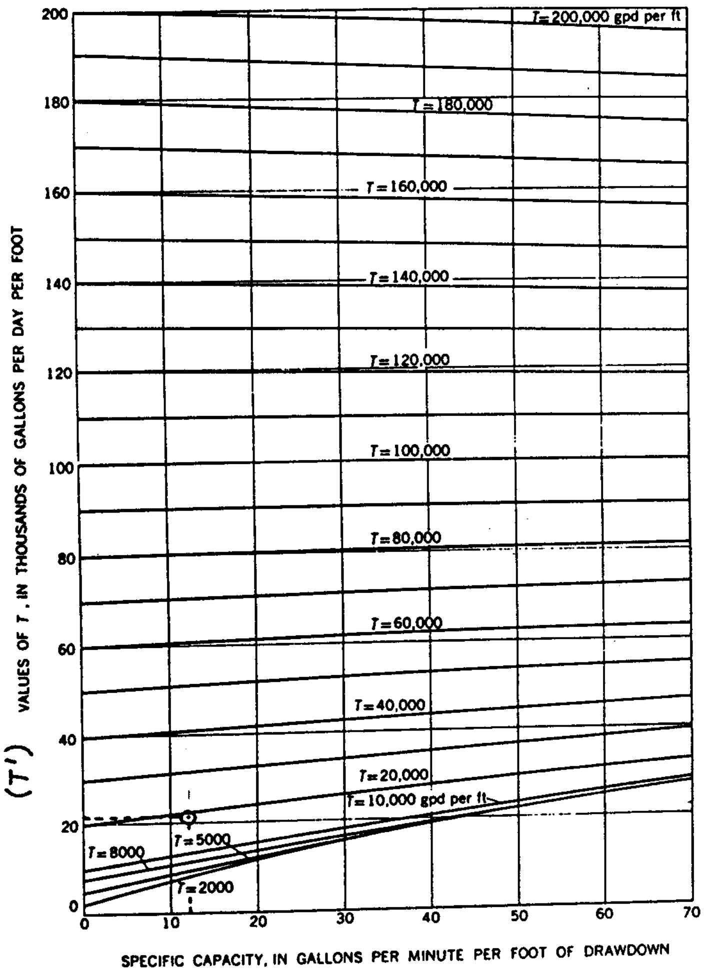

FIGURE 99.—Diagram for estimating the transmissibility of an aquifer from the specific capacity of a well.

the specific capacity of the pumped well can be substituted for the quantity Q/s for the appropriate distance from the well.

For small-diameter uncased wells that tap consolidated water-bearing rocks, or at least for wells that produced no sand or silt when developed, the distance r probably can be equated to the radius of the well. For instance, for a well 6 inches in diameter,

$$T' = C(1{,}684 - 264 \log_{10} 5S + 264 \log_{10} t),$$

in which

$$C = \frac{Q}{s} = \text{the specific capacity of the pumped well.}$$

In wells having perforated casing and for which no improvement in performance was noted upon development, some head is lost as the water moves through the perforations in the casing. The amount of head lost in this manner ranges widely according to whether or not the casing fits snugly against the wall of the hole. If it does, the drawdown within the aquifer at the wall of the hole presumably would be considerably less than within the well itself, and the specific capacity computed on the basis of the lesser drawdown would be considerably higher. An arbitrary increase, then, in the specific capacity probably would be justified for the computation of the coefficient of transmissibility. In consolidated formations in which the wall of a hole is rough and the casing does not fit tightly, the loss in head presumably is small and can be disregarded.

Many wells of large yield tap aquifers that consist of consolidated sand or gravel. Such wells yield readily to development and once they are developed the pumping level of the water both within and immediately outside the casing is generally higher than it would have been had they not been developed. It is difficult to estimate the extent to which the transmissibility of the materials in the immediate vicinity of a well has been increased by the development of the well. However, available data indicate that in many cases the effect is the same as if the well were 10 feet in diameter but had not been developed. Therefore, 996 (the factor for $r = 5$ ft) would be a reasonable value to substitute for K in the equation for T'.

Although many empirical data should be gathered as to the relation between the specific capacities of wells and the transmissibilities of the tapped aquifers before any final correlation is made, present knowledge seems to justify the following equation for wells that have a diameter of about 1 foot and that tap water-table aquifers consisting of unconsolidated sediments:

$$T' = C(1 \pm 0.3)(1{,}300 - 264 \log_{10} 5S + 264 \log_{10} t).$$

The factor (1 ± 0.3) should be adjusted upward for wells having a small diameter, for wells that are poorly developed, and for wells with poorly perforated casing, and downward for larger and well-developed wells.

ESTIMATING THE TRANSMISSIBILITY OF AN ARTESIAN AQUIFER FROM THE SPECIFIC CAPACITY OF A WELL

By Russell H. Brown

The use of figure 99 can be demonstrated by the following example. Assume that examination of well logs and related data has led to an estimate of 0.15 as a likely coefficient of storage, S, for a given water-table aquifer, that a review of well records has revealed a number of completion (or acceptance) tests, and that data taken from the best controlled test show, for a 30-hour pumping period, the specific capacity of a 6-inch well to be 12 gpm per ft of drawdown. The order of magnitude of the coefficient of transmissibility is to be determined. From the preceding discussion by Theis,

$$T'=\frac{Q}{s}(K-264\log_{10}5S+264\log_{10}t)$$
$$=12(1{,}684-264\log_{10}0.75+264\log_{10}1.25)$$
$$=12(1{,}684+33+26)$$
$$=20{,}900.$$

As shown by figure 99, the abscissa of $T'=20{,}900$ gdp per ft intersects the ordinate of specific capacity equals 12 gpd per ft of drawdown about where $T=19{,}000$ gpd per ft. If S should later prove to be 0.25 instead of 0.15, the revised value of T' would be 20,200 and, from the chart, T would be about 18,000 gpd per ft. Thus it is evident that even large differences in S do not materially affect the value of T and that exercising judgment in selecting a value for S will produce results of the correct order of magnitude.

As stated by Theis (p. 333), the formulas and related constants derived by him are not applicable to artesian conditions. The principal objection in attempting to extend their application from water-table conditions to artesian conditions is the large adjustment in the K factor that becomes necessary if, for example, $S=2\times10^{-4}$, which is one-thousandth the assumed $S=0.2$. However, a formula and set of constants for artesian conditions can be found by paralleling the Theis derivation and using an assumed coefficient of storage of 2×10^{-4}. If it is assumed again that $T=100{,}000$ gpd per ft, Theis' diagram (fig. 99) can be used without modification.

If $T=100{,}000$ gpd per ft and $S=2\times10^{-4}$, then from equation 1 on page 332

$$T=\frac{114.6Q}{s}\left[-0.577-\log_e\left(\frac{1.87r^2\cdot2\cdot10^{-4}}{100{,}000}\cdot\frac{S\cdot100{,}000}{2\cdot10^{-4}T}\cdot\frac{1}{t}\right)\right]$$
$$=\frac{114.6Q}{s}\left[-0.577-\log_e\left(\frac{(3.74r^2\cdot10^{-9})(5S\cdot10^3)}{(T\cdot10^{-5})t}\right)\right]$$
$$=\frac{66Q}{s}+\frac{264Q}{s}[-\log_{10}(3.74r^2\cdot10^{-9})$$
$$-\log_{10}(5S\cdot10^3)+\log_{10}(T\cdot10^{-5})+\log_{10}t].$$

Therefore,

$$T-\frac{264Q}{s}\log_{10}(T\cdot10^{-5})=-\frac{66Q}{s}+\frac{264Q}{s}$$
$$[-\log_{10}(3.74r^2\cdot10^{-9})-\log_{10}(5S\cdot10^3)+\log_{10}t].$$

Again let

$$T'=T-\frac{264Q}{s}\log_{10}(T\cdot10^{-5}).$$

Then

$$T'=-\frac{66Q}{s}+\frac{264Q}{s}[-\log_{10}(3.74r^2\cdot10^{-9})-\log_{10}(5S\cdot10^3)+\log_{10}t]$$
$$=\frac{Q}{s}[-66-264\log_{10}(3.74r^2\cdot10^{-9})-264\log_{10}(5S\cdot10^3)+264\log_{10}t].$$

Let

$$K=-66-264\log_{10}(3.73r^2\cdot10^{-9}). \qquad (5)$$

Then

$$T'=\frac{Q}{s}[K-264\log_{10}(5S\cdot10^3)+264\log_{10}t]. \qquad (6)$$

Values of K, computed for selected values of r, are as follows:

r (ft)	K	r (ft)	K
0.25	2,477	20	1,472
.50	2,318	30	1,379
1.0	2,159	40	1,313
5.0	1,790	50	1,262
10	1,633		

If the value of S is as large as 2×10^{-3} (10 times the assumed value) the effect will be to decrease K for $r=5$ feet by nearly 15 percent. For larger values of K this percentage obviously is lower, and for smaller values it is higher. Conversely, if S is as low as 2×10^{-5} (one tenth the assumed value) the effect will be to increase K by nearly 15 percent.

The application of the equation derived for T' for artesian conditions can be demonstrated by an example. Assume that the best estimate of S for a given artesian aquifer is 4×10^{-5}. Furthermore, data collected during a 30-hour acceptance test of a 6-inch well show that the specific capacity of the well is 7.5 gpm per ft of drawdown. The coefficient of transmissibility may be computed by following the same procedure used in the previous example.

Thus,

$$T'=\frac{Q}{s}[K-264\log_{10}(5S\cdot10^{3})+264\log_{10}t]$$
$$=7.5(2{,}477-264\log_{10}0.2+264\log_{10}1.25)$$
$$=7.5(2{,}477+184+26)$$
$$=20{,}200.$$

According to figure 99, $T=18{,}000$ gpd per ft (approx.) where the ordinate of 7.5 intersects the abcissa of 20,200. If it later develops that a value of 4×10^{-4} is a better estimate of S, then T' would be 18,200 and T would be about 16,000 gpd per ft.

A CHART RELATING WELL DIAMETER, SPECIFIC CAPACITY, AND THE COEFFICIENTS OF TRANSMISSIBILITY AND STORAGE

By Rex R. Meyer

The relationships of well diameter, specific capacity, and the coefficients of transmissibility, T, and storage, S, are shown graphically in figure 100. This graph was prepared by (1) computing, for various values of T and S, the theoretical drawdown in wells having diameters of 6, 12, and 24 inches, (2) computing the specific capacity of those wells (on the assumption that they are 100 percent efficient), and (3) plotting the specific capacity against S to form a family of curves which represent the different values of T. For the sake of clarity, the curves for a well 24 inches in diameter were not plotted in the upper part of the graph; they would be virtually parallel to the curves for a well 12 inches in diameter and lie above them at a distance equal to that between the curves for wells 6 inches and 12 inches in diameter. The specific capacity at the end of 1 day's pumping is shown on the left scale of the graph. The values of S, shown on the bottom scale, range from those for artesian conditions on the left to those for water-table conditions on the right. Each group of curves for a specific T is bracketed on the right margin.

Figure 100 can be used to determine the approximate T of an aquifer if the specific capacities of wells are the only available data. It also can be used to determine the approximate specific capacity of a well which is to be drilled into an aquifer for which T and S are known. The computed theoretical specific capacity is useful not only for planning purposes but also, when compared to the specific capacity determined from a field test, as a means of determining the approximate efficiency of a well. Although determinations made from figure 100 may not be exact owing to unknown factors that must be estimated, the graph serves as a measure for approximation.

A cursory study of the graph reveals that it has certain limitations. One of the principal factors affecting the specific capacity of a well is the entrance loss of the water. The graph is based on the assumption that the wells are 100 percent efficient or, in other words, that when the wells are pumped the water level inside and immediately outside the casing or screen is the same. Because, in most wells, the water level immediately outside is higher than inside, the observed specific capacity is somewhat less than that of an ideal well. The specific capacity of a well is affected also by the diameter of the well. The well diameters shown on the graph—6, 12, and 24 inches—are considered to be the effective diameters of the wells. If an aquifer is composed of consolidated rocks, the effective diameter probably is approximately the same as the diameter of the well. However, if the material in an aquifer consists of unconsolidated materials and if

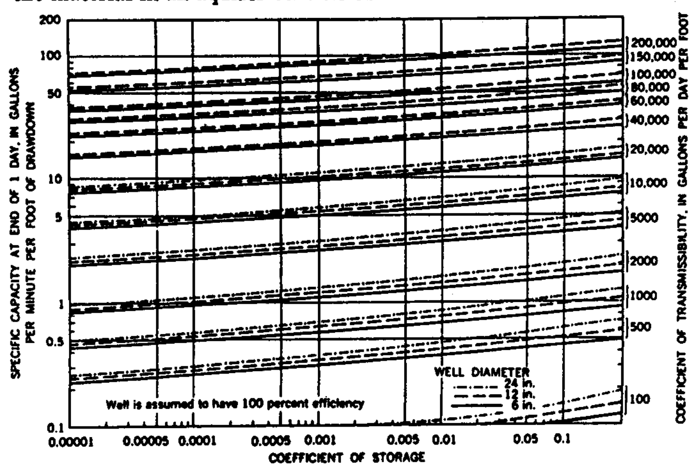

Figure 100.—Graph showing relation of well diameter, specific capacity, and coefficients of transmissibility and storage.

the well has been highly developed, the effective diameter may be substantially larger than the diameter of the screen. On the other hand, a seemingly highly developed well may be very inefficient because of caving or faulty construction, and, accordingly, have an effective diameter less than the diameter of the screen. Other conditions being the same, a change in the effective diameter has the greatest effect on the specific capacity of wells in aquifers that have a low T and a high S.

The graph shows that large changes in S correspond to relatively small changes in T and specific capacity; therefore, inaccuracy in estimating S generally is not a serious limiting factor. Moreover, from a general knowledge of the geology and hydrology, an aquifer usually can be classified as principally water table or artesian, and S can be estimated accordingly. However, the graph should not be used in an attempt to determine S even when accurate values of the specific capacity and T are available.

If the pumped well taps less than the full thickness of the aquifer—thus introducing vertical components of flow—or if it taps a thin water-table aquifer so that the water-level drawdown is a substantial fraction of the original saturated thickness, the graph obviously cannot be applied without serious error.

The time interval of 1 day used for computing the specific capacity scale on the graph was selected arbitrarily. An error will be introduced if the specific capacity determined in the field is based on a shorter or longer period of pumping. The amount of the error is small for high values of T and low values of S but increases substantially for low values of T and high values of S.

The procedure for using the log graph to determine T from the specific capacity of a well is as follows:

1. Select the specific capacity on the left margin.
2. Move horizontally along the abcissa to the intersection of the ordinate through the estimated value of S.
3. From this intersection move along a curve or parallel to the family of curves, and find the value of T on the right margin.

Although the specific capacity at the end of 1 day's pumping can be computed for an ideal well tapping an aquifer having known values of T and S, it can be determined more easily and quickly from the graph. To determine the theoretical specific capacity of such a well, the procedure described above is reversed; move left along or parallel to the curve from the known value of T to the intersection of the ordinate through the known value of S; thence move horizontally to the left margin and read the specific capacity.

If the graph is used with an understanding of its limitations, it should provide a useful tool in ground-water studies.

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