WATER WELL TESTING IN THE ATHABASCA OIL SANDS, COMPLEMENTARY GUIDANCE

Submitted To:

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Scientific Summary

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Project Description: The objective of this project was to develop complimentary guidance on water well testing being conducted in the Alberta Oil Sands region. It provides supplementary information to existing guidance found in Alberta Environment and Sustainable Resource Development's *Guide to Groundwater Authorization* report (2011).

Project Deliverables: Final Guideline Report

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Project Status: Final Report Submitted

Highlights/Milestones/Key Findings/Etc.: The main body of this report provides complementary guidance on key elements of the execution, interpretation and reporting of pumping tests. Section 2 outlines appropriate practice for the design and execution of pumping tests. Section 3 provides a discussion on practice for the processing of pumping test data. Section 4 outlines an approach for interpreting pumping tests. Section 5 presents an overview of common techniques for estimating the potential long-term yield of a pumping well. Section 6 gives the reader guidance on reporting the results of a pumping well test. There are also four appendices in the report which provide additional supplemental information. Appendix 1 contains a checklist for executing pumping tests. Appendix 2 displays plots of a series of diagnostic responses to pumping. Appendix 3 provides the reader a "bridge" between the disciplines of hydrogeology and petroleum engineering, a review of the terminology and foundations of pumping test analyses from the perspectives of both disciplines. Finally, Appendix 4 presents a case study of the application of the approaches for interpreting pumping tests and estimating the long-term yield of a well in buried-valley aquifer system that is typical of the northern Great Plains.

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EXECUTIVE SUMMARY

This report has been prepared to provide guidance for the execution and interpretation of pumping tests in Alberta. The report is intended to complement the ***Guide to Groundwater Authorization*** (Alberta Environment, 2011). The ***Guide to Groundwater Authorization*** contains only general guidance regarding the interpretation of pumping tests. The guidance regarding the estimation of the theoretical long-term yield of a pumping well also includes recommended calculations. However, no details are provided regarding the interpretation of key quantities specified in the calculations. The present report provides guidance on the following key elements of the execution, interpretation and reporting of pumping tests:

- A discussion of appropriate practice for the design and execution of pumping tests (Section 2);
- A discussion of appropriate practice for the processing of pumping test data (Section 3);
- Presentation of an approach for the interpretation of pumping tests (Section 4);
- A discussion of techniques for estimating the potential long-term yield of a pumping well (Section 5); and
- Recommendations for the appropriate reporting of pumping tests (Section 6).

Many pumping tests are conducted in Alberta to support oil and gas development. To provide a "bridge" between the disciplines of hydrogeology and petroleum engineering, a review of the terminology and foundations of pumping test analyses from the perspectives of both disciplines are presented in Appendix 3.

The application of the approaches for interpreting pumping tests and estimating the long-term yield of a well in buried-valley aquifer system that is typical of the northern Great Plains is illustrated in Appendix 4.

1. INTRODUCTION

Pumping tests are a foundational technique of hydrogeology and it is impossible to exaggerate the importance of their reliable interpretation in groundwater applications. Applications in which reliable interpretations of pumping tests are crucial include analyses of regional groundwater flow, evaluations of groundwater resources and predictions of the effects of developing additional groundwater supplies. Reliable interpretations of pumping tests are also essential for the design of measures to intercept groundwater inflows to excavations and for depressurizing aquifers beneath excavations. The results of pumping tests also guide the design of remedial measures at contaminated sites.

A key document for the interpretation of pumping tests in Alberta is the Alberta Environment ***Guide to Groundwater Authorization*** (March 2011). The ***Guide to Groundwater Authorization*** contains valuable guidance, some of which is quite specific. For example, detailed recommendations are provided regarding the frequency of water level monitoring and the minimum duration of pumping tests. In contrast, the guidance regarding the interpretation of pumping tests is very general. It is indicated only that the selection of the aquifer test method is to be based on the hydrogeology of the proposed test site. The guidance regarding the estimation of the theoretical long-term yield of a pumping well also includes recommended calculations. However, no details are provided regarding the interpretation of key quantities specified in the calculations.

The report you are reading has been prepared to provide guidance for the execution and interpretation of pumping tests in the Athabasca Oil Sands of Alberta. An attempt is made to achieve this objective by complementing the ***Guide to Groundwater Authorization*** with:

- A discussion of appropriate practice for the design and execution of pumping tests (Section 2);
- A discussion of appropriate practice for the processing of pumping test data (Section 3);
- Presentation of an approach for the interpretation of pumping tests (Section 4);
- A discussion of techniques for estimating the potential long-term yield of a pumping well (Section 5); and
- Recommendations for the appropriate reporting of pumping tests (Section 6).

Application of the approaches for interpreting pumping tests and estimating the long-term yield of a well in buried-valley aquifer system that is typical of the northern Great Plains is illustrated in Appendix 4.

This report has been prepared by and for hydrogeologists. However, many of the pumping tests in Alberta are conducted to support the development of energy resources, and the results of the tests may be reviewed by specialists in petroleum engineering. The petroleum engineering literature is a treasure trove of information on pumping tests. Hydrogeologists have frequently been slow to take advantage of the developments in petroleum engineering pressure transient testing. To broaden the perspective of hydrogeologists, and to provide a common basis for discussions between hydrogeologists and petroleum engineers, the development of the foundations of pumping test methods in both disciplines is included in Appendix 3.

2. RECOMMENDED PRACTICE FOR THE DESIGN AND EXECUTION OF PUMPING TESTS

Overview

The key to a successful pumping test is careful planning. All pumping tests should be designed, and the design should be documented in a work plan. This section includes a review of the objectives of a pumping test, a presentation of the elements of a pumping test design and a checklist designed to assist in the execution of reliable pumping tests.

2.1 Introduction

When designing a pumping test, it is important to start with a clear understanding of the purpose of the well and the motivation for conducting a pumping test on it.

Pumping wells may be installed for a variety of purposes. These purposes may include:

- Fundamental site characterization;
- Inference of hydrogeologic structures;
- Development of short and long-term water supplies;
- Land drainage (dewatering);
- Control of groundwater inflows into excavations;
- Depressurization of confined aquifers;
- Containment of contaminated groundwater; and
- Removal of dissolved contaminants from groundwater systems.

Pumping tests are conducted to accomplish a variety of objectives, and the way a test is conducted and interpreted should be consistent with those objectives. At the most fundamental level, a pumping test is conducted to observe how the groundwater system responds to a controlled stress. In an area where relatively little information is available, it may be appropriate to begin with the simple objective of developing initial insights into the general characteristics and properties of a site. Progressing to quantitative analyses, short-term pumping tests are frequently sufficient to estimate the hydraulic properties of an aquifer (transmissivity, vertical hydraulic conductivity, storage coefficients, leakage coefficients). A short-term test may also be sufficient to support an estimate of the short-term yield of a production well in anticipation of designing a test that will assist in assessing the long-term response of a groundwater system.

To infer the presence of boundaries and to estimate the locations of the boundaries, it may be necessary to conduct a pumping test for an extended period of time, with multiple observation wells in appropriate locations. Tests of extended duration may also be required if the objectives of a test include obtaining insights regarding the long-term yield of an aquifer, assessment of the potential interference with neighboring groundwater supplies and assessment of potential impacts on surface water features.

When considering developing groundwater supplies to support the development of natural resources, it is frequently only feasible to conduct relatively brief tests with a limited number of observation wells. Scaling-up the results from such a test will necessarily involve extrapolation. Under these conditions, it is important to bear in mind that the results of a pumping test are provisional. Ongoing monitoring will be essential to confirm the assumptions of the original analysis or revise the site conceptual model.

2.2 The Elements of a Pumping Test Design

At a minimum, it is recommended that the work plan include the following elements.

- An indication of the regulatory context in which the test is being conducted.
- A clear statement of the objectives of the pumping test.
- Confirmation that the pumping well is suitable.
- A description of the techniques that will be used to control and measure the discharge.
- A description of the frequency of water level and flow rate measurements.
- A description of the techniques that will be used to measure changes in water levels, and to ensure that these measurements are reliable.
- A description of redundancy measures for measuring the discharge and water levels.
- A description of the approach that will be adopted to identify the changes in water levels that are due only to pumping.
- A preliminary indication of the likely pumping rate for the test and the duration of pumping.
- Documentation of the design of a step test.
- A description of the approach that will be adopted to confirm that the expected pumping rate can be sustained over the planned duration of the test.
- An indication of duration of the monitoring of recovery after the end of pumping.
- A summary of the wells that will be monitored during the test, including their distances from the pumping well and the elevations of the screens of the wells.
- Predictions of the likely magnitudes and timing of the water level changes that will be caused by pumping.

<u>Assessing the suitability of a pumping well</u>

It is important to appreciate the details of a pumping well to assess whether it is appropriate for testing. When assessing a well, it is necessary to confirm that the well has been designed appropriately. Key questions that must be answered include:

- Have the filter pack and screen been designed following accepted hydrogeologic practice? In practice, this means: Has the well been designed following the procedures described in **Groundwater and Wells** (Driscoll, 1986; Sterrett, 2007) and Hanna (2006)?
- If one of the objectives of the test is to infer the vertical hydraulic conductivity, is the degree of penetration of the well screen appropriate? and
- Has the well been developed adequately? To answer this question it is necessary to consider the method that was used to drill the well, and the techniques and level of effort involved in developing the well.

On the requirement that the pumping rate be constant

The literature on the interpretation of pumping tests is generally cast in terms of *constant-rate pumping tests*, it is important to note that there is no theoretical reason why the discharge rate during a pumping test must be constant. The computer-assisted interpretation techniques that are used widely by hydrogeologists use the principle of superposition to generalize all linear aquifer models for time-varying pumping. Superposition is illustrated in Figure 2-1, in which the drawdowns generated with a complex pumping history are matched with a theoretical solution.

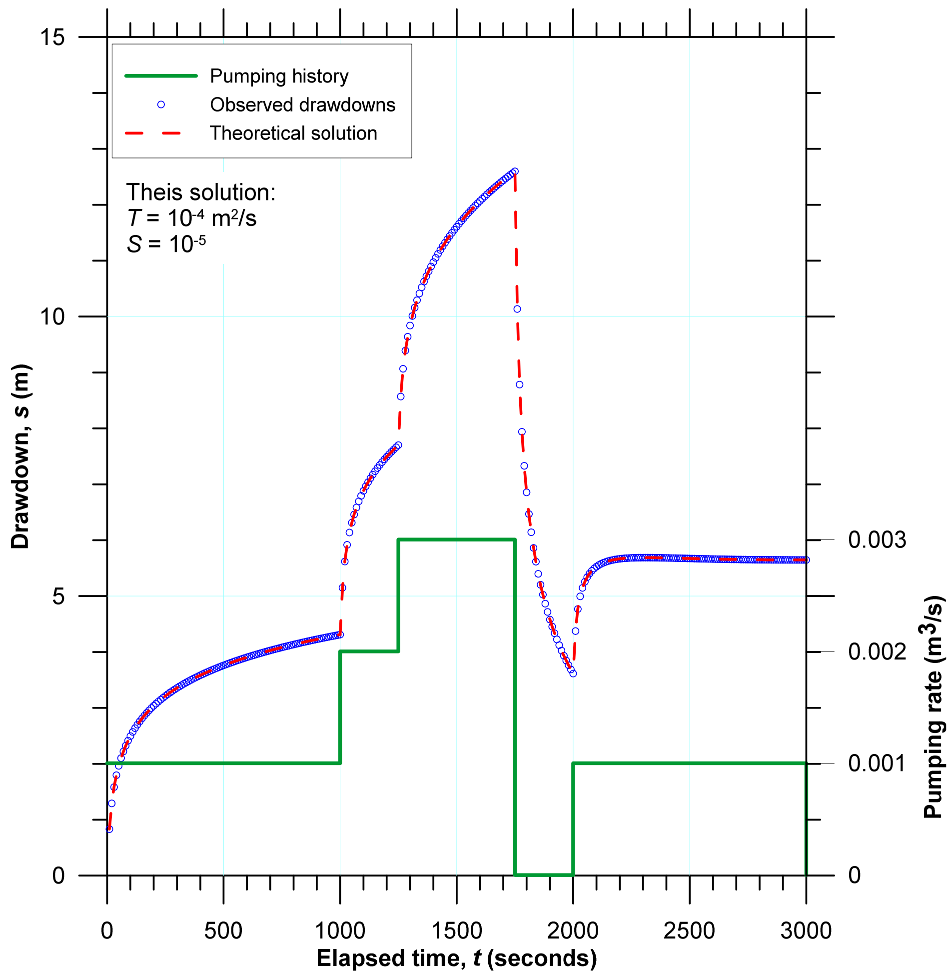

Figure 2-1. Example pumping test analysis for a complex pumping history.

Despite the fact that there may not be any theoretical reason why the pumping rate must be constant, we recommend that to the greatest extent possible, the pumping rate should be held constant during a test. Requiring that the pumping rate be held constant ensures that the pumping rate is *controlled and monitored*. In contrast, a significant variation in the pumping rate during a test is frequently an indication that the pumping rate has not been controlled adequately. Adopting a constant pumping rate also simplifies the detection of boundary effects and the plotting of the drawdown derivative.

There may be circumstances under which it is challenging to maintain a constant pumping rate. Power surges and outages are not uncommon, particularly in remote settings. In Alberta, the exsolution of gas may give rise to apparent fluctuations in pumping rates and groundwater levels. Pumping rates will vary during almost every test that extends beyond a few hours. It is therefore essential that water levels and pumping rates be measured frequently and reliably. The best approach for reliable monitoring of water levels is to use transducers with dataloggers, with regular manual monitoring with electric water level tapes where feasible. The best approach for reliable monitoring of pumping rates is to use totalizing flowmeters that are equipped with dataloggers, with period checking of in-line flowmeter measurements.

Prediction of the likely effects of pumping

The inclusion in the work plan of expectations regarding the effects of pumping will require developing predictions of the drawdowns that are likely to be observed in the pumping well and at available observation wells. This is important for several reasons. First, the planning should include some assessment of whether a particular observation well will provide clear-cut indications of the effects of pumping. Second, the likely magnitude of water level changes should guide the selection of pressure transducers. Pressure transducers vary according to their range and accuracy; the selection of an appropriate transducer requires estimation of the magnitudes of the drawdown.

Computer-assisted interpretation packages that are in general use in hydrogeologic practice are ideally suited to assist in the planning of pumping tests. These packages are generally used to infer aquifer properties from observations made during the test (pumping rates and changes in groundwater levels that are due only to pumping). However, they can also be used to predict changes in water levels, with the analyst specifying the pumping rate and aquifer structure and properties. The prediction of pumping test response is one application of a general approach that is referred to as *forward modeling*.

In many locations in Alberta, abundant regional-scale information is available to inform forward modeling that can support the design of pumping tests. This information includes:

- Mapping of the surficial and bedrock geology conducted by the Alberta Research Council (Now available from the Alberta Geological Survey, http://www.ags.gov.ab.ca/publications/pubs.aspx?series=map);
- Mapping of the regional hydrogeology of Alberta conducted by the Alberta Research Council (Now available from the Alberta Geological Survey, http://www.ags.gov.ab.ca/publications/abstracts/DIG_2009_0003.html);
- Regional geological and hydrogeological reports available from the Alberta Geological Survey (see for example, http://www.ags.gov.ab.ca/groundwater/index.html); and
- Regional groundwater assessments conducted by Hydrogeological Consultants Ltd. (http://www.hcl.ca/reports.asp).

3. RECOMMENDED PRACTICE FOR THE PROCESSING OF PUMPING TEST DATA

Overview

The first question that the interpreter of a pumping test must ask is: "Are the data worth interpreting?" Pumping test interpreters must assure themselves that the data are reliable; there is little point in trying to analyze data that are known to be dubious. Pumping test interpreters must also assure themselves that the data are interpretable. In practice, this means that it must be feasible to isolate the changes in groundwater levels caused by pumping from outside influences. Many processes may give rise to changes in groundwater levels, including seasonal trends, fluctuations in barometric pressure, pumping from other wells and changes in water levels in nearby surface water features. Most analyses are now undertaken with computer-assisted methods. The implicit assumption in these methods is that the changes in groundwater levels are caused only by pumping. Distinguishing between the effects of pumping and effects of outside influences is the responsibility of the hydrogeologist, not the analysis software.

3.1 Data Quality Assurance

Prior to conducting any analyses, the pumping test interpreter must accomplish and document the following data quality assurance tasks:

- Confirmation that the pumping rate has been controlled and measured correctly; and
- Confirmation that the water level measurements are reliable.

A complete report of a pumping test must include documentation of the methods that have been used to control and measure the pumping rate. The reporting should plot the "spot" readings from the instantaneous flowmeter and the cumulative volume pumped as recorded with the totalizing flowmeter. Example plots of flowmeter records from the same pumping test are shown in Figures 3-1 and 3-2. The documentation should also include the results of the checking of the totalizing flowmeter.

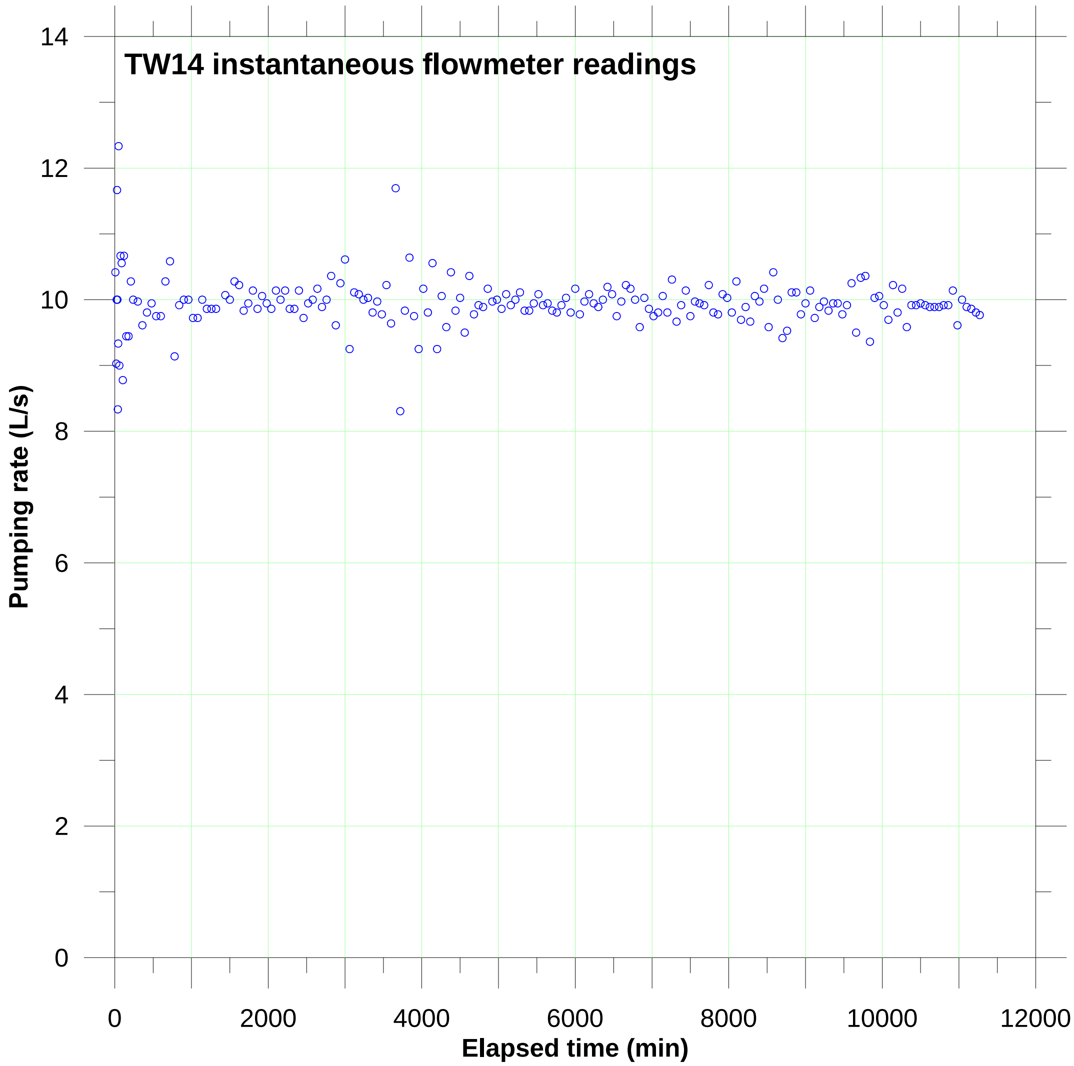

Figure 3-1. Spot flowmeter measurements.

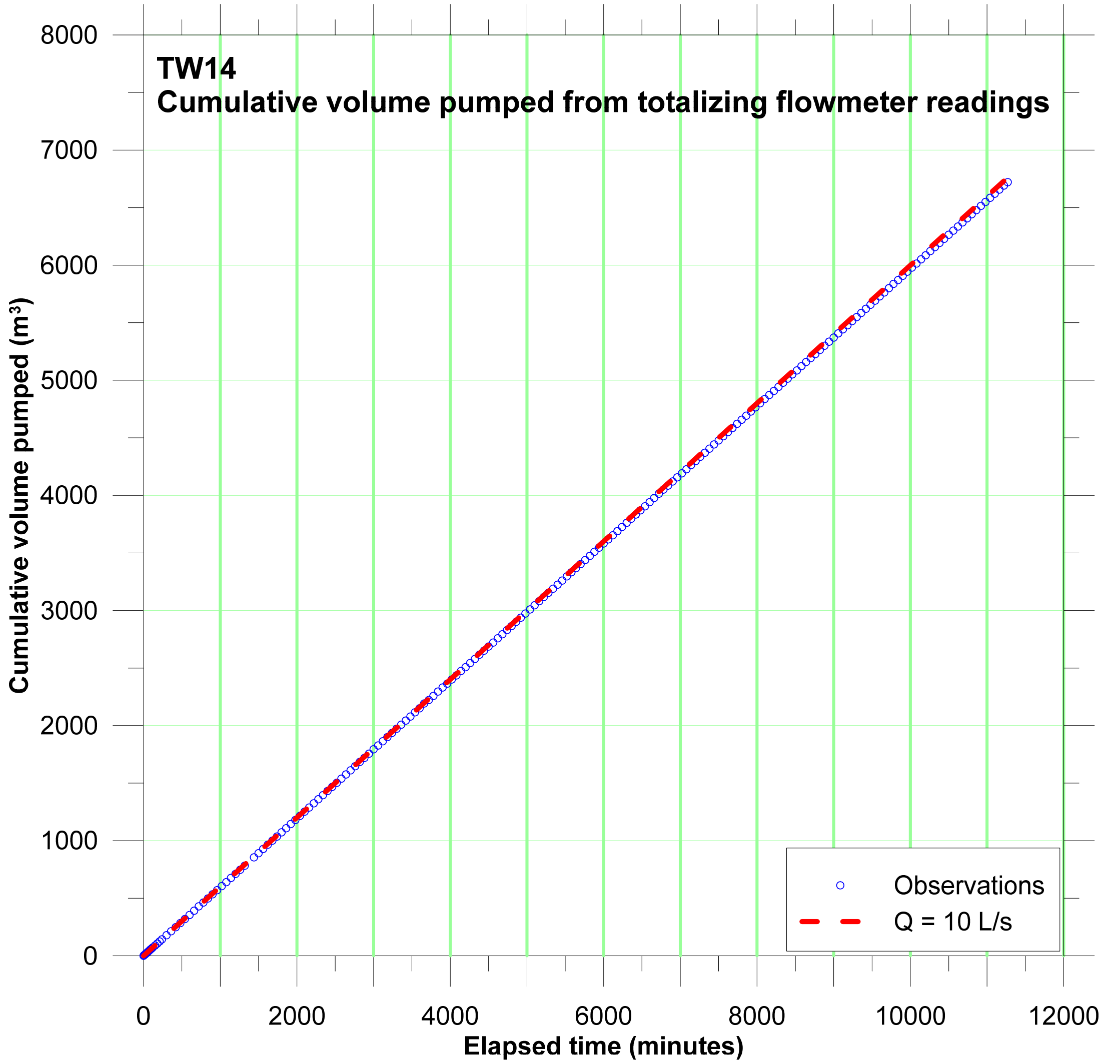

Figure 3-2. Cumulative volume pumped as recorded with the totalizing flowmeter.

A complete report of a pumping test must also include documentation of the methods that have been used to measure water levels. It is now standard practice to use transducers and dataloggers to measure groundwater levels. Keys to the successful use of transducers include the selection of an appropriate transducer, understanding how fluctuations in barometric pressure affect the transducer, and checking of the transducer in the field.

Transducers differ in the range of submergence over which they operate effectively and the accuracy of the measurements within that range. To select an appropriate transducer for a particular test, analyses must be conducted in advance of the pumping test to develop an appreciation of the conditions that are likely to be encountered.

Transducers also differ in the way they are affected by changes in barometric pressure. For non-vented diaphragm transducers, a barometer is required to account for differences between the barometric pressure in the field and the barometric pressure in the factory at the time the transducer was sealed. In contrast, for vented transducers the atmospheric pressure is the same on both sides of the diaphragm. However, it is important to understand that changes in barometric pressure give rise to changes in groundwater levels, regardless of whether the transducers are non-vented or vented. Changes in groundwater levels caused by fluctuations in barometric pressure should be quantified for most pumping tests. In practice, this requires that barometric pressure be monitored with a transducer dedicated to the continuous measurement of atmospheric pressure.

Transducers are calibrated in the manufacturer's laboratory, but the laboratory is generally not within walking distance of the pumping test. It must be shown in the field that that a known change in the submergence is equal to the change reported with the datalogger. If the transducer fails this check, it should not be used.

Since transducers may drift and sometimes fail outright, it is also important that the documentation include cross-comparisons of water levels recorded with the transducers and water levels recorded with an electric water level tape. The cross-comparison can be as simple as a hydrograph of the continuous water level measurements on which the manual measurements are superimposed, as shown in Figure 3-3.

For pumping of extended duration, and tests conducted in remote settings, it may not be feasible to assemble a record of manual measurements that is sufficient to confirm the reliability of the transducer measurements. Under these circumstances, it may be necessary to incorporate some redundancy in the monitoring program, by installing multiple transducers at key locations.

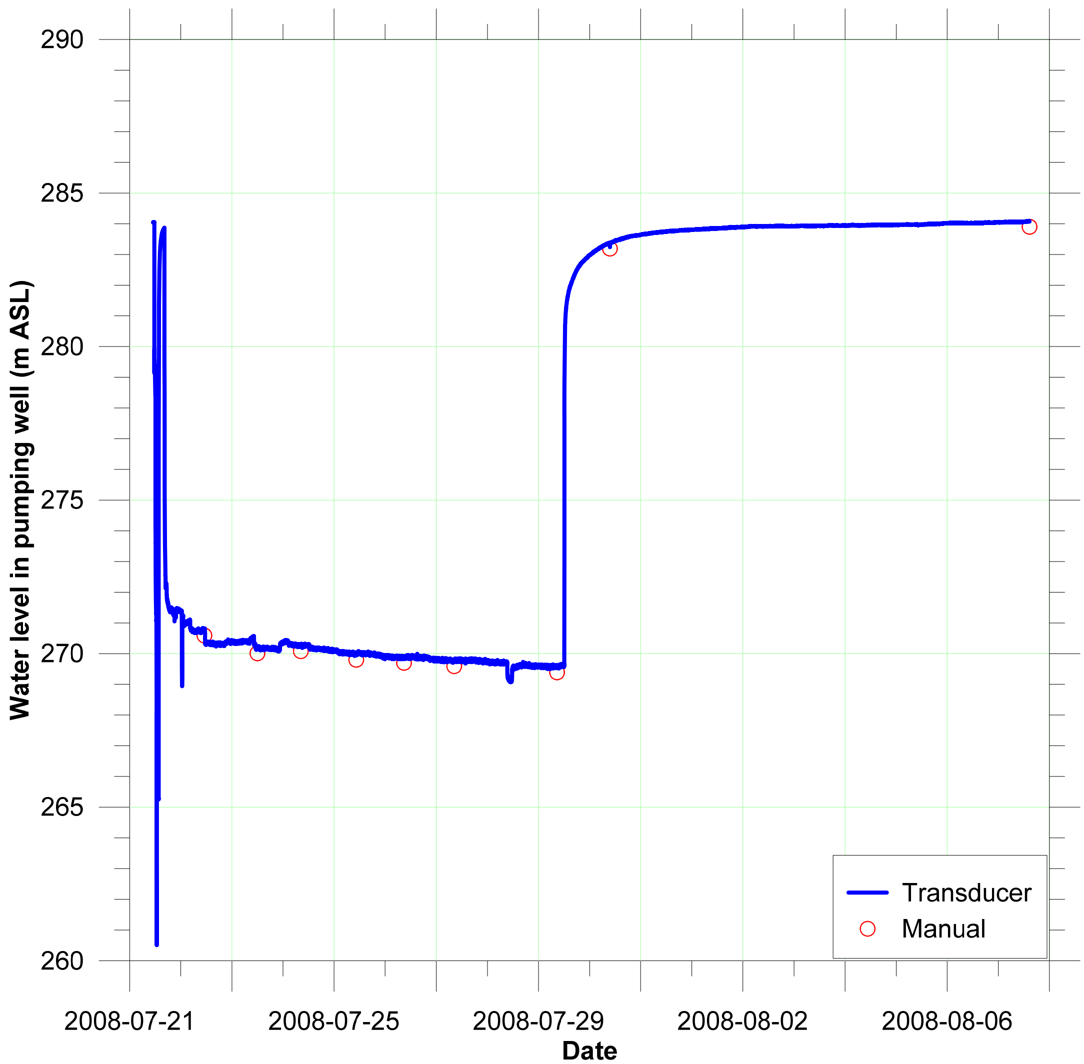

Figure 3-3. Comparison of continuous and manual water level.

3.2 Identification of Water Level Changes Caused Only by Pumping

All aquifer interpretation methods, whether they make use of manual calculations and/or computer-assisted interpretation packages, are founded on the assumption that all water level changes are caused only by pumping. The measure of the change in the water level is referred to as the *drawdown*. Before any analysis is conducted, the water level data must be processed to remove all changes in water levels that are not due to pumping. There are many sources of changes in water levels in aquifers. These sources include fluctuations in barometric pressure. It is important to note that two corrections for barometric fluctuations may be required. If non-vented transducers are used to measure water levels, the raw data will have to be corrected to yield the correct submergence of the transducer. Fluctuations in barometric pressure will also give rise to changes in water levels in the aquifer, and should be accounted for if the fluctuations are significant. The barometric pressure must always be measured during a pumping test.

Other sources of water level changes include fluctuations in water levels in overlying confining units, fluctuations in water levels in surface water features that are connected to the aquifer, diurnal fluctuations associated with phreatophytes, and short and long-term background trends in water levels. The processing of the data must also account for the effects of pumping from wells other than the test production well.

An error that is incorporated relatively frequently in the processing of pumping test data is the assumption that the drawdown s, at any distance r and time t is defined as:

$$s(r,t) = h(r,t=0) - h(r,t)$$

Here $h(r,t=0)$ denotes the groundwater level at the start of pumping and $h(r,t)$ denotes the water level at any subsequent time.

The correct definition of the drawdown is:

$$s(r,t) = h_0(r,t) - h(r,t)$$

Here $h_0(r,t)$ denotes the groundwater level that would have been observed at an elapsed time t if there had not been any pumping.

A background trend in effect represents a moving datum from which changes in water levels are calculated. When there is a background trend it water levels it is necessary to observe or synthesize a record of water levels that reflect the background conditions but not the effects of pumping. This is fundamentally different from using recovery data to estimate the likely water level at the start of pumping. The data from a pumping test conducted adjacent to a river in which the water level fluctuated during the test are used to illustrate this subtle concept. The calculation of the incorrect drawdown is illustrated in Figure 3-4. In this case, the water level records are sufficiently long to confirm that observation wells PW-4-118, MW-5-100 and MW-20-120 are reliable indicators of the water levels that would have been observed at PW-4-85 in the absence of pumping. The calculation of the correct drawdown is illustrated in Figure 3-5.

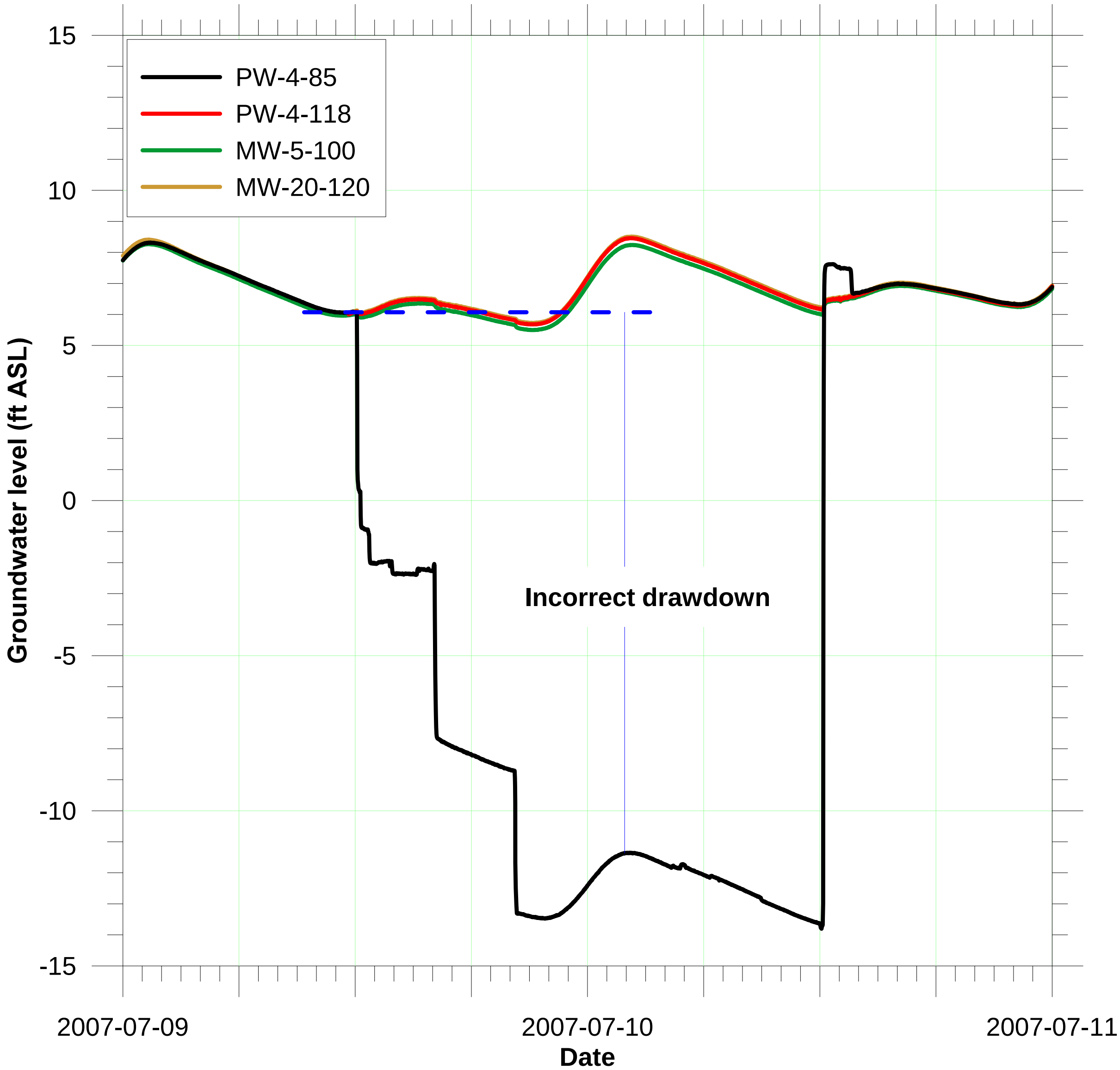

Figure 3-4. Incorrect interpretation of drawdowns during a complex pumping test.

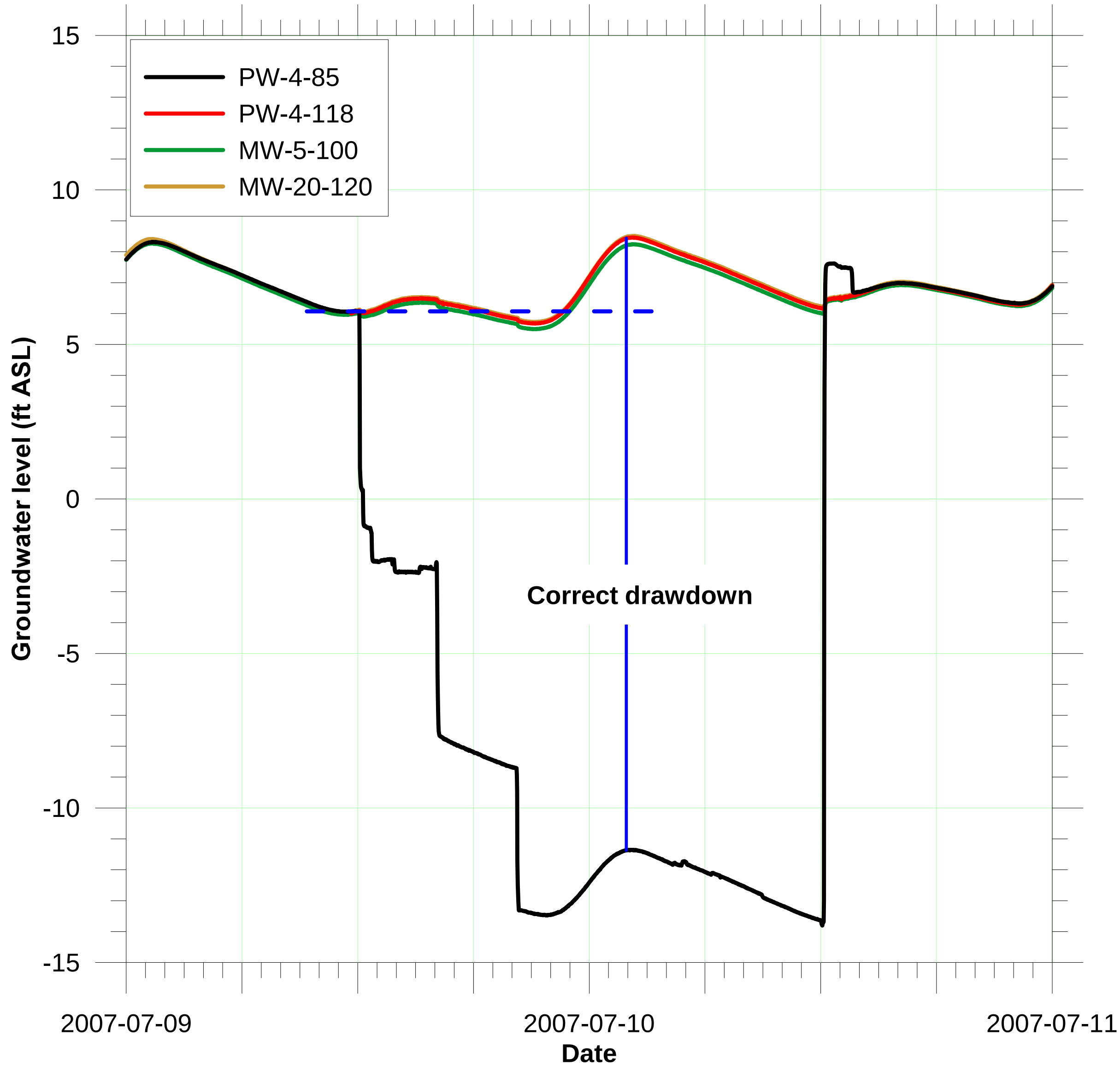

Figure 3-5. Correct interpretation of drawdowns during a complex pumping test.

4. A SUGGESTED APPROACH FOR THE INTERPRETATION OF PUMPING TESTS

Overview

The interpretation of a pumping test is conceived of as a progression of steps from the preliminary estimation of aquifer properties with simple models, to the inference of the conceptual model and estimation of representative aquifer properties, to finally the prediction of long-term performance. In this section an approach will be suggested for moving through this progression. The section is concluded by a decision tree to assist in the interpretation. A set of diagnostic plots to assist analysts in inferring the appropriate conceptual model for refined analyses is included in Appendix 1.

Each of the analyses described in this section are consistent with "classical" approaches. However, many of these methods appear to have been abandoned in favor of moving immediately to complex interpretation approaches. Therefore, the motivation for each of the analyses is presented. One point that will be emphasized here is that it appears many hydrogeologists pay little attention to the hydraulics of pumping wells. The drawdowns in the pumping well itself provide some of the best information about the system.

4.1 Introduction

In a general sense, the interpretation of pumping tests consists of correlating well flows with changes in groundwater pressures or water levels, and drawing inferences about the ability of an aquifer to transmit and store water (Narasimhan, 1978). The prominence of pumping tests in hydrogeologic practice reflects the fact that when they are interpreted appropriately, they provide *in situ* information about an aquifer on a scale that is meaningful for long-term management of water resources.

In practice, the interpretation of pumping tests consists of matching the results of a model to the observed changes in groundwater levels that have been attributed to pumping. Traditionally, the models have been theoretical solutions for idealized systems. There is a rich literature of these solutions (see for example, Hantush, 1964; Stallman, 1971; Weeks, 1978, Batu, 1998). Until the late 1980s, the parameters of the models were estimated with graphical methods, referred to collectively as type-curve analyses. Step-by-step instructions for accomplishing particular pumping test analyses with type curves are presented in the primary sources and in widely accessible monographs (for example, Kruseman and de Ridder, 1990). Similar methods are described in petroleum engineering reference manuals, including the classic work of Earlougher (1977).

Computer-assisted aquifer test interpretation packages were introduced in hydrogeologic practice in the late 1980s. These packages are now used widely in hydrogeologic practice and have largely supplanted manual type-curve analysis. In addition to being more convenient than manual methods, the computer-assisted interpretation packages offer several other advantages. For example, these packages include the support for more complex conceptual models and more general geometries than can be handled with type-curve methods.

This section does not include a review of the many models that are available to interpret a pumping test. The references cited above serve this purpose. Instead, emphasis is directed to those aspects of pumping test interpretation that are generally not addressed: a phased approach towards estimating the aquifer properties, inference of the conceptual model through the use of diagnostic plots; and the development of internally consistent interpretation of drawdown data from multiple observation wells and/or pumping wells.

4.2 Guiding Principles For The Interpretation Of Pumping Tests

The interpretations of pumping tests should be guided by five principles.

1. Principle 1: Build up in complexity.
   A phased approach should be adopted for the analysis, starting with a simple conceptual model and introducing complexity gradually.

2. Principle 2: Diagnose the aquifer response.
   Multiple plotting approaches and derivative analysis should be used to infer the structure of the aquifer from the response to pumping.

3. Principle 3: Take advantage of recovery data.
   Recovery data should be analyzed to check the interpretations of the drawdown data, and to extend the effective duration of pumping.

4. Principle 4: Seek internal consistency.
   The parameter estimates that are developed should be internally consistent.

5. Principle 5: Include a reality check.
   The interpretations should be consistent with everything else that is known about the aquifer.

4.3 A Phased Approach for the Interpretation of Pumping Tests

The interpretation of a pumping test should begin as simply as possible and build up in complexity. We caution against proceeding with any analysis that aims to include as much complexity as possible from the outset. Rather, it is good hydrogeologic practice to start any analysis with a rough estimate of the "right" answer, and then seek to refine that estimate. We recommend that for cases in which all of the data are available, the interpretation follow a systematic sequence of analyses that is outlined below.

1. Preliminary interpretation of the results of step testing using the Hantush-Bierschenk analysis of the pumping well drawdowns.

2. Development of a first-cut estimate of the transmissivity from the specific capacity of the pumping well.

3. Estimation of the aquifer and well parameters from the complete record of pumping well drawdowns during the step test.

4. Check on the interpretations of the step test with the pumping well drawdowns observed during the constant-rate pumping test.

5. Simplified estimation of the transmissivity from the pumping well drawdowns during the constant-rate pumping test.

6. Estimation of transmissivity from the drawdowns of the period of "ideal" aquifer response.

7. Diagnosis of the complete response to pumping.

4.3.1 *Hantush-Bierschenk Analysis of the Pumping Well Drawdowns from a Step Test*

It has been recommended previously that all constant-rate pumping tests be preceded by step tests. A step test provides data on the pumping rate that can be supported for a constant-rate test. Just as importantly, a step test provides a basis for accounting for that portion of the drawdown that is not due to head losses in the formation. The results of the step test analysis can then be used to adjust the drawdowns observed in a pumping well so that they can be treated the same way as the data from other observation wells. Although the analyses are approximate, they are presented in detail as they are both useful and not applied sufficiently frequently in practice.

The observations during the step test are plotted in Figure 4-1. The data from the test are used to illustrate the development of rough estimates of the nonlinear well losses and the transmissivity, following the approaches of Hantush (1964) and Bierschenk (1964).

The test comprised four 90-minute pumping steps, indicated by the red line (right axis). The blue line denotes the corresponding drawdown observed in the pumping well (left axis). For the purposes of a preliminary analysis, it is assumed that the water level in the pumping well is nearly stable by the end of each 90-minute step. Inspection of Figure 4-1 suggests that this is a reasonable approximation for the first three steps. It is important to keep the results of the Hantush-Bierschenk analysis in perspective; when drawdowns do not stabilize by the end of each step, the fitting coefficients should be regarded only as first-cut estimates.

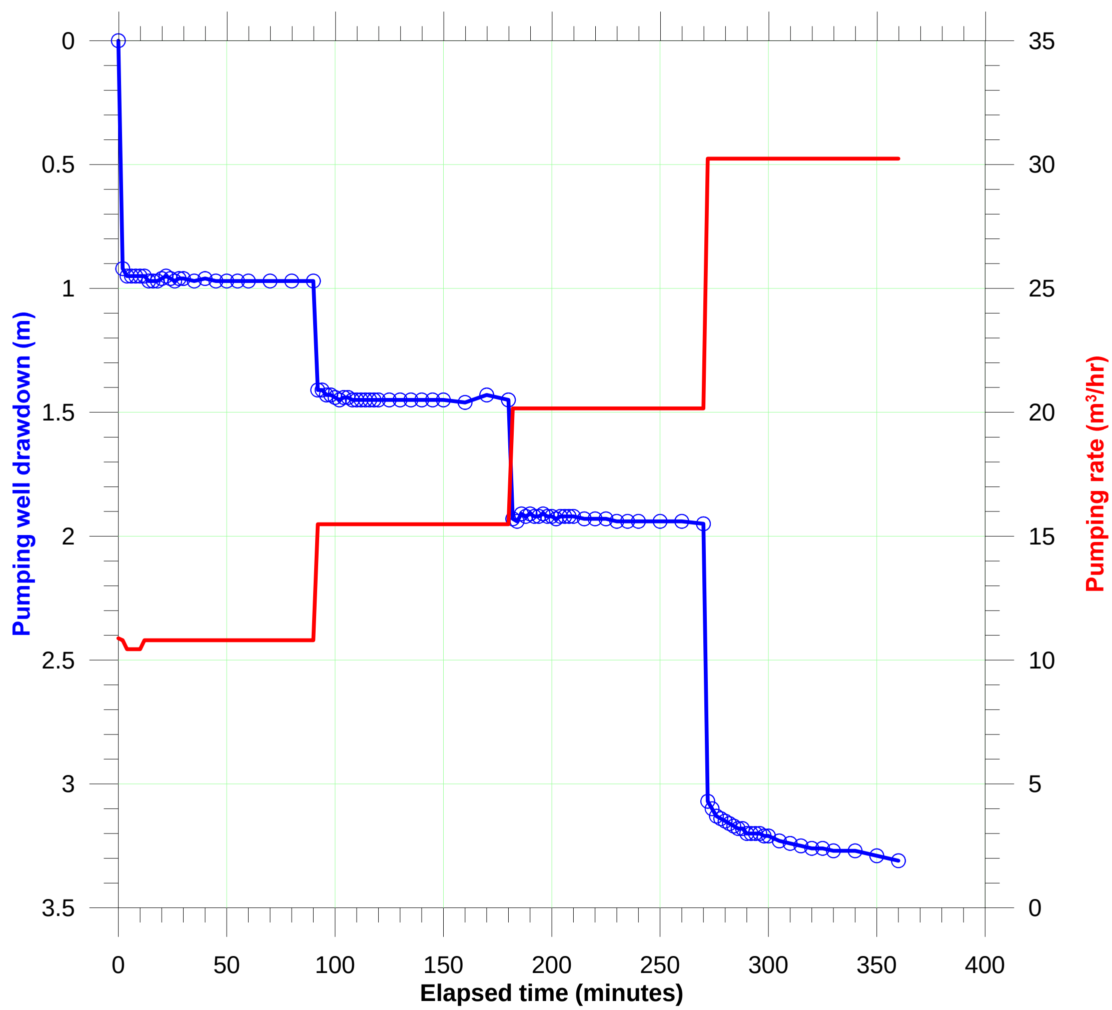

Figure 4-1. Data from a step test.

The results from the end of each step are summarized below. The average pumping rate during each step is denoted by *Q*; s_w represents the drawdown at the end of each step. The quantity s_w/Q is the *specific drawdown* at the end of each step. The specific drawdowns for the step test are plotted against the pumping rate in Figure 4-2.

Pumping step	Average pumping rate (m^3/hr)	Drawdown at end of step (m)	Specific drawdown (m/m^3/hr)
0	0.00	0.00	
1	10.75	0.97	0.0902
2	15.48	1.45	0.0937
3	20.16	1.95	0.0967
4	30.24	3.31	0.1095

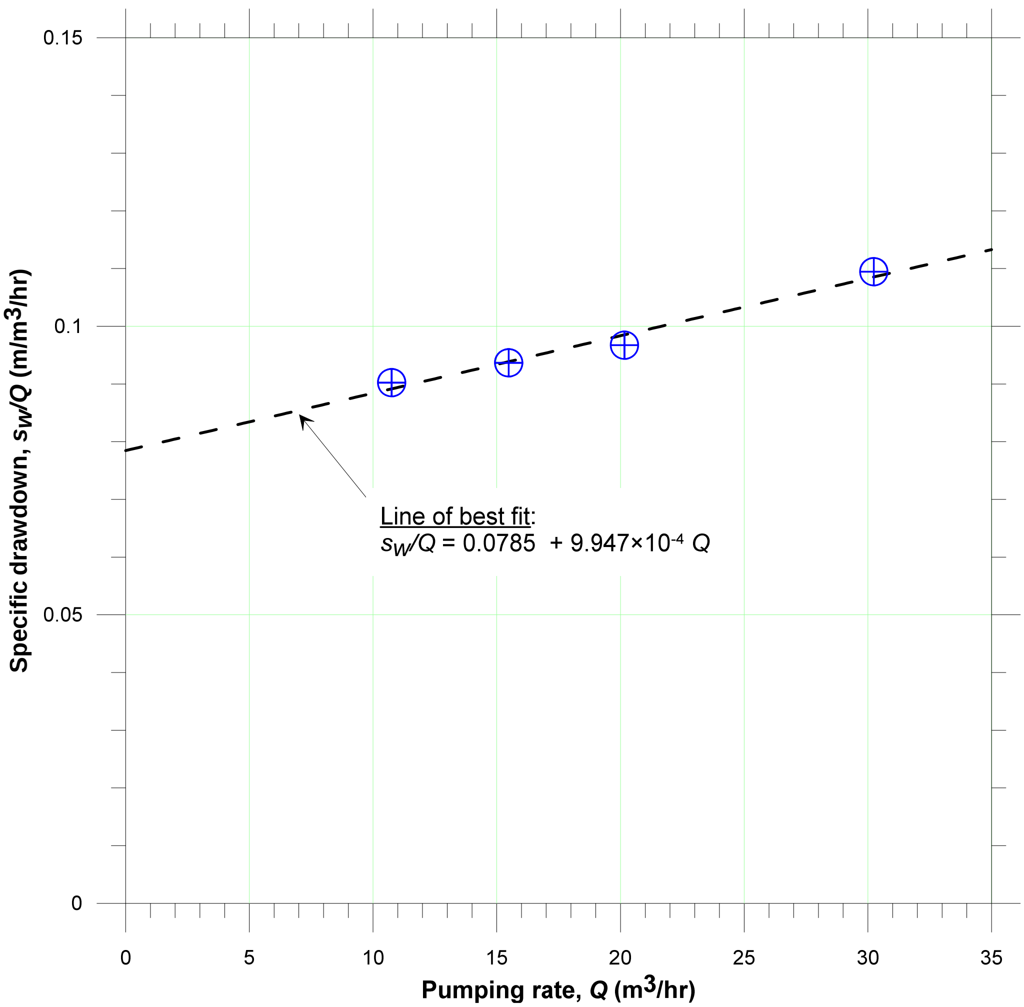

Figure 4-2. Hantush-Bierschenk plot of specific drawdowns

The specific drawdowns at the end of each step plotted against the pumping rate approximate a straight line; this suggests that as an approximation, the drawdowns at the end of each step can be approximated with the Jacob (1947) model:

$$s_w = BQ + CQ^2$$

The coefficients estimated from the line of best fit are:

- B = 0.0785 m/m^3/hr; and
- C = 9.947×10^{-4} m/(m^3/hr)^2.

As a check on the calculations, in Figure 4-3 the pumping rate for each drawdown as predicted with the fit obtained with the Hantush-Bierschenk analysis is superimposed on the observations. The relatively close match confirms that the fitting is appropriate.

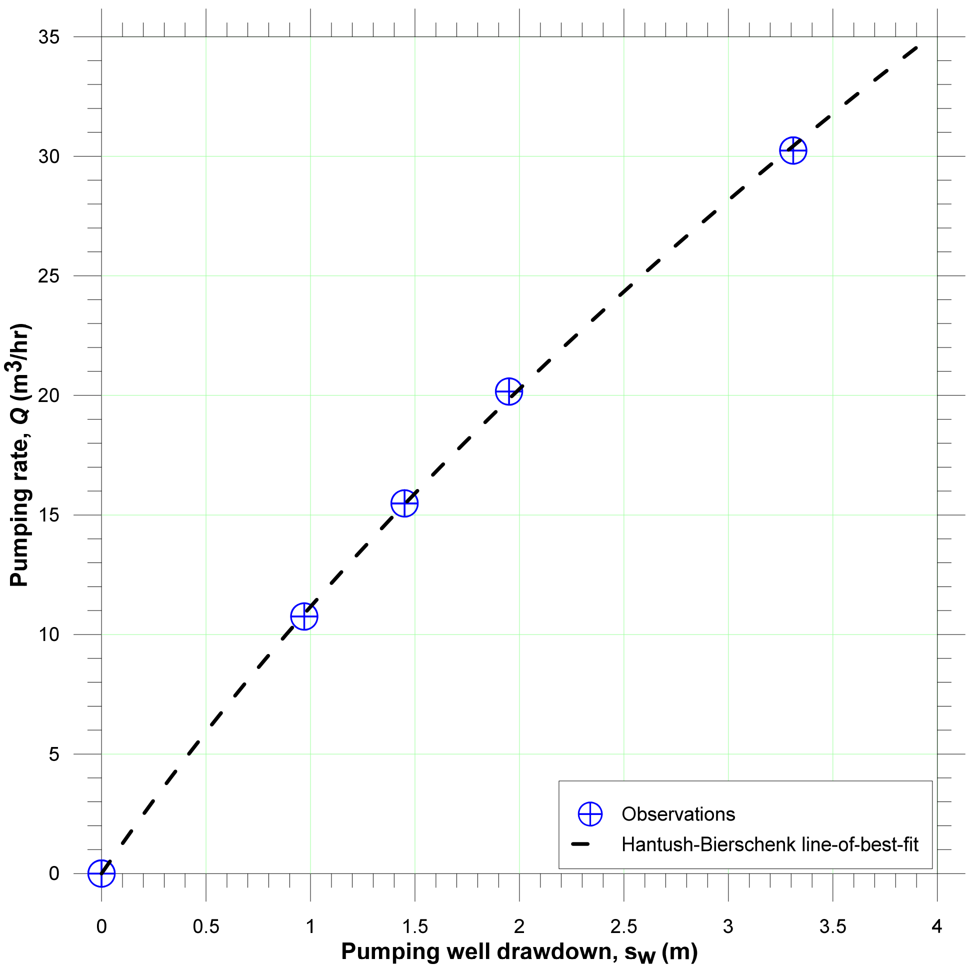

Figure 4-3. Pumping rate versus pumping well drawdown.

4.3.2 Development of a First-Cut Estimate of the Transmissivity from the Specific Capacity of the Pumping Well

The results of the Hantush-Bierschenk analysis can be extended to develop a first-cut estimate of the transmissivity.

The *specific capacity* is defined as the ratio of the pumping rate and the pumping well drawdown:

$$SC = \frac{Q}{s_w}$$

Following the work of Theis and others (1963) and Walton (1970), Driscoll (1986) has suggested that as a first approximation, the transmissivity for a confined aquifer can be estimated from the specific capacity as:

$$T \sim 1.3\ SC$$

Here the transmissivity, *T,* and the specific capacity, *SC,* are specified in consistent units.

The coefficient *B* of the Jacob (1947) model corresponds to the reciprocal of the specific capacity with the nonlinear well losses removed. For the step test data plotted in Figure 4-2, the specific capacity with nonlinear well losses removed is given by:

$$SC = \frac{1}{B} = \frac{1}{0.0785\ m/m^3/hr} = 12.7\ m^3/hr/m$$

Therefore, as a first approximation, the transmissivity is:

$$T \sim 1.3\ SC = \ 1.3 \times 12.7\ m^3/hr/m \approx 16.5\ m^2/hr$$ or **<u>400 m²/d</u>**.

It is important to note that when estimating the transmissivity with this approach, it is assumed implicitly that the pumping well drawdowns are due <u>only</u> to linear flow in the formation. This is an important assumption. The preceding Hantush-Bierschenk analysis effectively separates the linear and nonlinear well losses; however, the analysis does not distinguish between the sources of the linear head losses. In particular, the analysis neglects the potential effects of a zone of disturbed material around the pumping well and the additional head losses associated with partially penetrating wells. Bearing this limitation in mind, the transmissivity value is qualified as a first-cut estimate. More refined analyses may reveal that the first-cut estimate is not representative of the large-scale properties of the aquifer. Rather than pointing to a flaw in the analyses, significant differences with the results of refined analyses may provide important insights into conditions in the immediate vicinity of the pumping well.

4.3.3 Estimation of the Aquifer And Well Parameters from the Complete Record of Pumping Well Drawdowns During a Step Test

When a complete set of observations is available for a step test, it is recommended that a more refined analysis of the step test be conducted. This next level of analysis involves matching the entire time-drawdown record with an idealized conceptual model. The hydraulics of pumping wells generally does not receive much attention in hydrogeologic practice. We use the discussion of the interpretation of pumping well drawdowns as an opportunity to discuss in some detail each of the sources of the drawdown in a pumping well. The discussion has important implications with respect to not only the interpretation of aquifer properties, but also to the interpretation of the efficiency and long-term yield of a well.

Accounting for variations in the pumping rate, skin losses and additional nonlinear well losses, the drawdown in the pumping well is written as:

$$s_w(t;n) = s_f(t;n) + \Delta s_{skin} + \Delta s_{nonlinear}$$

The drawdown in the pumping well at any time *t* is the sum of three components:

- Head losses in the formation, $s_f(t;n)$;
- Additional head losses across a skin zone, Δs_{skin}; and
- Additional nonlinear losses, $\Delta s_{nonlinear}$.

Drawdown due to head losses in the formation

The first part of the solution represents the drawdown due to head losses in the formation, evaluated at a radial distance corresponding to the radius of the well (r_w is specified as the borehole radius). It is recommended that an extended version of the Theis (1935) model be adopted for the analysis:

$$s_f(t;n) = \frac{1}{4\pi T}\sum_{i=1}^{n(t)} \Delta Q_i \, W\left(\frac{r_w^2 S}{4T(t - ts_i)}\right)$$

The Theis solution is generalized for time-varying pumping with the principle of superposition. *W* denotes the Theis well function, *n(t)* represents the number of pumping steps that have occurred up to the current time *t*, and the terms ts_i and ΔQ_i represent the starting time and increment of the pumping rate of the i^{th} pumping step, respectively. These quantities are illustrated in Figure 4-4.

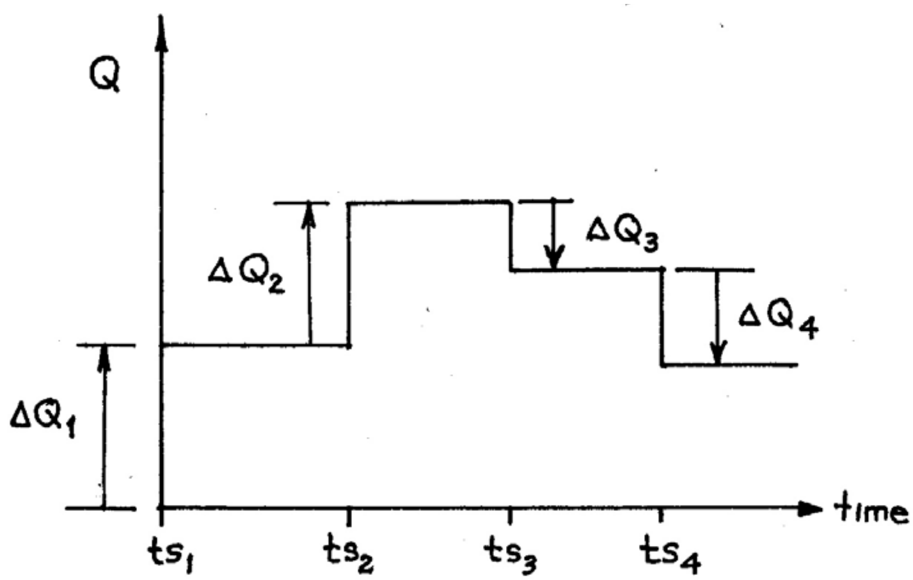

Figure 4-4. Application of the principle of superposition for time varying pumping.

Additional head losses across a skin zone

The second component of the pumping well drawdowns is referred to additional skin losses. The additional skin losses are illustrated schematically in Figure 4-5. Regardless of how carefully a well is drilled, and how vigorously the well is developed after installation, there is always the possibility that a zone of disturbed material may be created around it. The zone of disturbed material is referred to as the "skin", and the additional head losses due to its presence are referred to as a "skin effect". Skin effects may arise from the use of drilling mud in porous media, or from the sealing of fractures in rock with rock flour. Skin effects may be mitigated to a certain extent by proper well development following drilling.

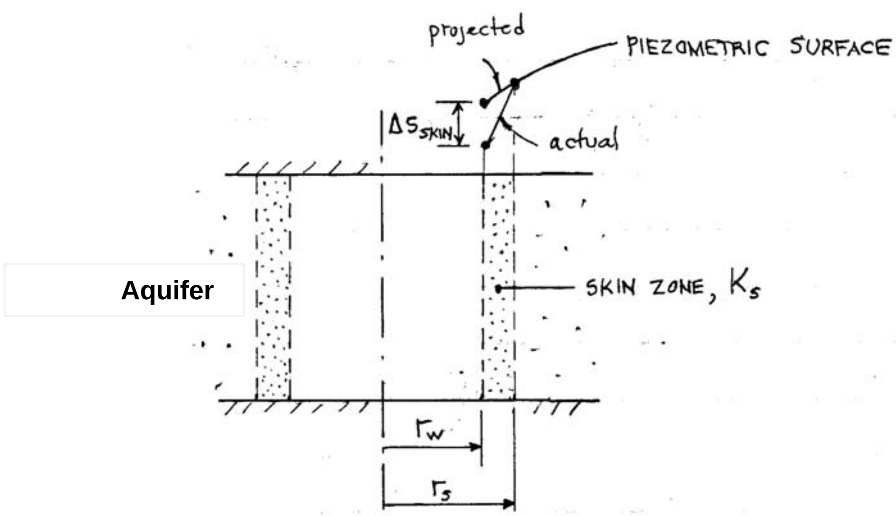

Figure 4-5. Cross-section through a well that is surrounded by a skin.

The additional decline in the piezometric surface condition illustrated schematically in Figure 4-5 corresponds to a condition referred to as "positive skin". A positive skin condition arises when the material immediately surrounding the pumping well are less permeable than the aquifer. Under some circumstances, the installation and development of the well may cause the zone around the well to become more permeable than the aquifer. This is referred to as a "negative skin" condition. A negative skin condition gives rise to a flattening of the drawdown cone around the well.

The expression for the skin losses follows the approach of Ramey (1982c):

$$\Delta s_{skin}(Q_n) = \frac{Q_n}{4\pi T} 2S_w$$

Here S_w is designated the dimensionless skin loss coefficient; Q_n corresponds to the pumping rate at the current time *t*. The current pumping rate is related to the steps according to:

$$Q_n = \sum_{i=1}^{n(t)} \Delta Q_i$$

Although the skin effect has typically been interpreted as a process that gives rise to an additional drawdown, conditions may arise in which the drawdown in the vicinity of the pumping well is actually reduced relative to projection of the piezometric surface to the outside edge of the well. For example, during development, the fine-grained fraction may be removed from the sediments that surround a well. The possibility that the skin effect may reflect either a decrease or an increase of the hydraulic conductivity around a well is reflected in the definition of the dimensionless skin factor presented by Hawkins (1956):

$$S_w = \left(\frac{K - K_s}{K_s}\right) \ln\left\{\frac{r_s}{r_w}\right\}$$

Here K and K_s are the hydraulic conductivities of the formation and the skin, respectively, and r_s is the radius of the skin. If the hydraulic conductivity of the skin zone is reduced with respect to the formation, the value of S_w is positive (*positive* skin), and if the hydraulic conductivity of the skin zone is decreased with respect to the formation, the value of S_w is negative (*negative* skin). In practice, neither the extent of the skin zone nor its hydraulic conductivity can be known. Instead, the parameter S_w is treated as a lumped quantity that is estimated as part of the analysis.

Additional head losses occur when a pumping well does not penetrate the full thickness of an aquifer. The conceptual model of a partially penetrating well is illustrated in Figure 4-6.

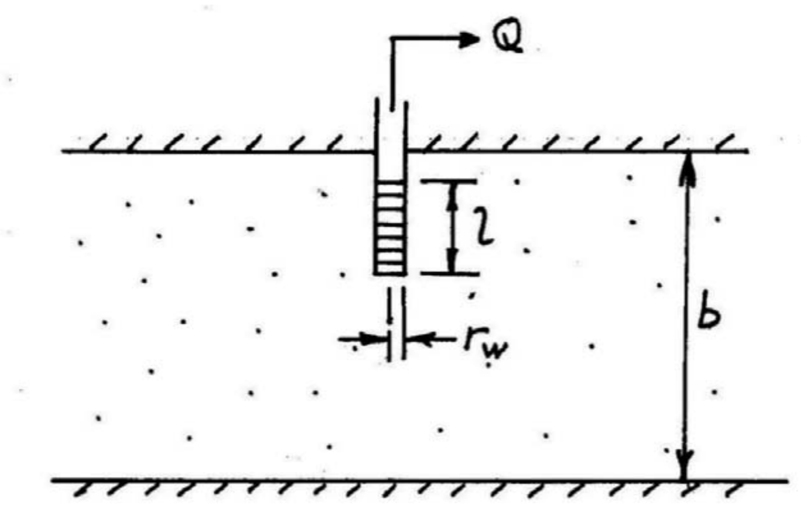

Figure 4-6. Conceptual model for a partially penetrating well.

Rigorous analyses of flow to partially penetrating pumping wells suggest that the additional drawdowns caused by partial penetration are established relatively quickly, and are directly proportional to the pumping rate (Hantush, 1961). Therefore, the additional drawdown caused by partial penetration has the same general form as skin losses. The losses due to partial penetration are written in terms of a *pseudo-skin coefficient,* S_{pp}:

$$\Delta s_{pp} = \frac{Q}{4\pi T} 2S_{pp}$$

Several approaches have been developed to estimate the additional head losses due to partial penetration. Brons and Marting (1961) developed a simple approach that approximates closely the results obtained with more elaborate calculations:

$$S_{pp} = \left(\frac{b-l}{l}\right)\left[\ln\left\{\frac{b}{r_w}\right\} - G\left(\frac{l}{b}\right)\right]$$

Here b is the aquifer thickness, l is the length of the well screen, and $G\left(l/b\right)$ is a function tabulated in Brons and Marting (1961). Bradbury and Rothschild (1985) used regression to develop the following functional form from the tabulated values of G:

$$G\left(l/b\right) \cong 2.948 - 7.363\left(\frac{l}{b}\right) + 11.447\left(\frac{l}{b}\right)^2 - 4.675\left(\frac{l}{b}\right)^3$$

It is important to note that it is not possible to distinguish between the additional drawdowns arising from an actual skin zone from the additional head losses that arise because of partial penetration. This is because the additional head losses have similar characteristics and are modeled with the same general form. Therefore, the skin factor inferred from the analysis of the pumping well drawdowns provides a lumped representation of the additional drawdowns that are proportional to the pumping rate. Analysts must use their understanding of the way the well was drilled, constructed and develop to assess the most likely cause of the additional linear head losses.

Additional nonlinear losses

The additional nonlinear losses represent the combined effects of nonlinear flow processes in the formation close to the well and turbulence within the well itself. For a well that is screened across unconsolidated materials, there should not be any nonlinear losses in the formation if the well has been designed properly, following the guidelines in **Groundwater and Wells** (Driscoll, 1986; Sterrett, 2007). However, nonlinear head losses may arise from turbulence within the well itself as flow constricts around equipment in the well, or due to friction losses along the well casing if the distance between the well screen and the pump intake is relatively long.

The additional nonlinear well losses for the nth pumping step are represented as:

$$\Delta s_{nonlinear} = CQ_n^P$$

Here *C* and *P* represent the nonlinear well loss coefficient and nonlinear well loss exponent, respectively. Rorabaugh (1953) suggested the use of the exponent *P* as a generalization of the Jacob (1947) model discussed in Section 4.3.1.

Recommended practice for the interpretation of step tests

In practice, the complete records of drawdowns for each step test are matched with the solution obtained by combining the drawdown components:

$$s_w(t) = \frac{1}{4\pi T}\sum_{i=1}^{n(t)} \Delta Q_i\, W\left(\frac{r_w^2 S}{4T(t-ts_i)}\right) + \frac{Q_n}{4\pi T} 2S_w + CQ_n^P$$

The parameters may be estimated with a nonlinear least squares fitting routine as implemented in a computer–assisted interpretation package. The parameters that *can* be adjusted to achieve a match to the pumping well drawdowns are *T*, *S*, S_w, *C*, and *P*. However, it is not possible to estimate a unique set of parameters from the fitting. We recommend that only the transmissivity, *T*, and the skin loss coefficient S_w be adjusted. The appropriate treatment of the other parameters is described below.

Nonlinear well loss coefficient *C* and exponent *P*

We recommend that the value of *C* be fixed from the Hantush-Bierschenk analysis. Values of the exponent *P* presented in the literature are usually close to 2, and the analysis does not deteriorate if the exponent is fixed at 2, thereby eliminating another fitting parameter in the analysis.

Storage coefficient, *S*

The responses observed during single-well tests are generally not sensitive to the storage coefficient. In consequence, it is generally not possible to obtain a reliable estimate of S from the analysis of a step test. We recommend a two-step analysis approach.

Analysis Step 1:

In the first step, the drawdowns are matched treating S as an adjustable parameter and assuming that there are no skin effects, setting $S_w = 0.0$. If a relatively good match to the drawdowns is achieved with a value of the storage coefficient that is typical for sand and gravel aquifers, the analysis can stop here. Typical values of the storage coefficient for confined aquifers range from 10^{-5} to 10^{-3} (Lohman, 1972; Boonstra, 1989). Estimated values of S that are beyond this range will suggest that there are skin effects.

Analysis Step 2:

In the second step, when skin effects are suspected, the storage coefficient is fixed at a value that is representative of typical sand and gravel aquifers, 10^{-4}, and the skin loss coefficient S_w is treated as an adjustable parameter.

The two-step analysis approach is illustrated with the results from the step test that were presented previously in Figure 4-1.

Analysis Step 1

The results of the first analysis, treating both the transmissivity and the storage coefficient as adjustable parameters, are shown in Figure 4-7. The nonlinear well loss coefficient and exponent, *C* and *P*, are fixed at the values estimated from the Hantush-Bierschenk analysis (noting the conversion of the time units). A good match is obtained to the entire drawdown record; however, the fitted storage coefficient, 3.2×10^{-17} is <u>not</u> physically realistic. In reporting these parameter estimates, it would be noted that obtaining an implausible estimate of the storage coefficient indicates that there is a flaw in the analysis. In particular, the results suggest that the model that has been invoked does not capture all of the processes that give rise to the drawdowns in the pumping well.

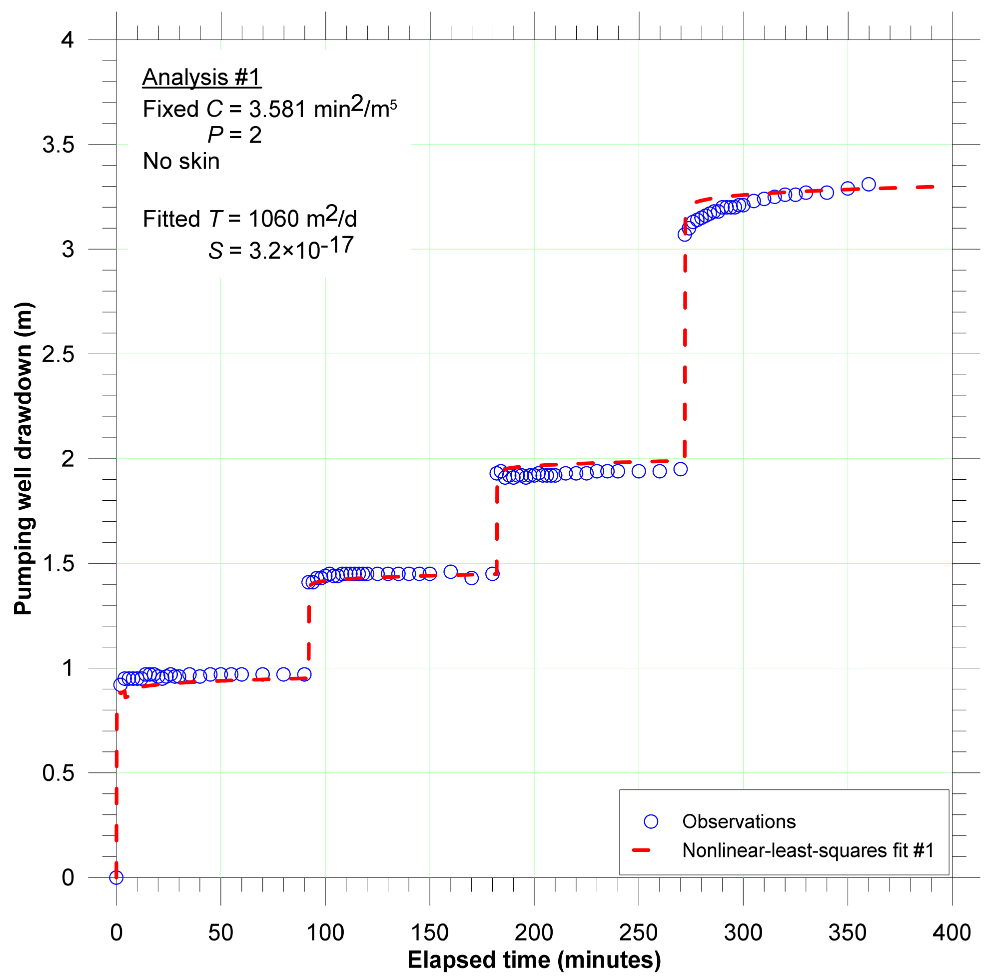

Figure 4-7. Results of fitting of step test – Analysis #1.

Analysis Step 2

For a second analysis, the storage coefficient is fixed at a physically realistic value of 10^{-4} and the nonlinear well loss coefficient and exponent, *C* and *P*, are fixed at the values estimated from the Hantush-Bierschenk analysis. The dimensionless skin factor, S_w, is added as an adjustable parameter. The results of a second nonlinear least-squares fit are shown in Figure 4-8. A good match is again obtained to the entire drawdown record. The estimated transmissivity is not significantly different from Analysis Step 1; however, a relatively large skin factor is required to achieve a match if the analysis is constrained to incorporate a realistic value of storage coefficient. The results of the analysis suggest that at the end of the test, head losses in the formation accounted for 0.81 m of the total drawdown of 3.31 m, the nonlinear head losses accounted for 0.91 m, and additional head losses across a skin zone accounted for 1.60 m. The head losses in the aquifer account for about 25% of the total drawdown in the pumping well.

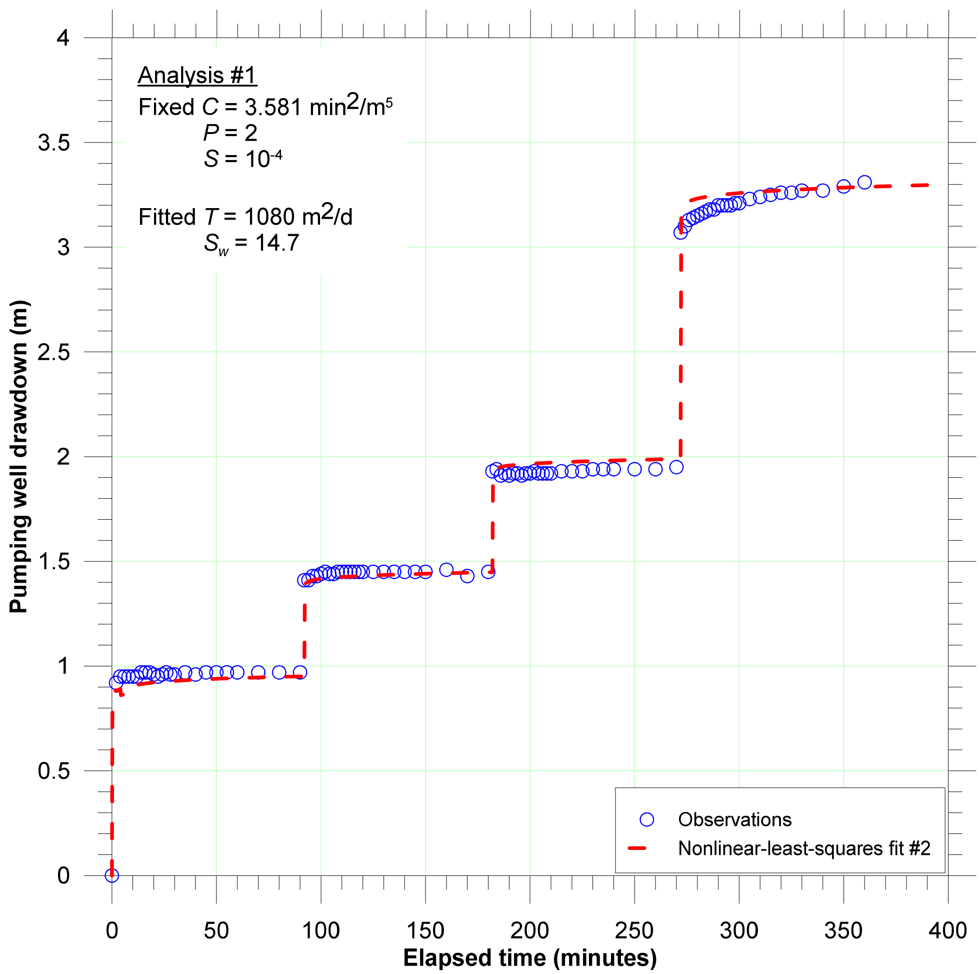

Figure 4-8. Results of fitting of step test – Analysis #2.

Assessment

The first-cut estimate of the transmissivity, developed in Section 4.3.2, was 400 m^2/d. In contrast, the transmissivity estimated from transient analyses of the complete drawdown record is about 1,100 m^2/d. The fundamental limitation of the first-cut analysis is that it does not distinguish between the sources of the linear portion of the drawdown, that is, between head losses in the formation and head losses across a skin zone. The first of the analyses of the complete drawdown records yielded a non-physical storage coefficient. There is no way a value of $S = 3.2\times10^{-17}$ should be reported without an indication that it is non-physical. The analysis is nevertheless useful, as obtaining non-physical parameter estimates has important diagnostic value. The second analysis is not quite complete. The analyst would be expected to examine the method of well construction and the well details to present a likely source of the relatively large skin factor. In this case, the well was drilled with a mud rotary rig, and the well may not have been developed sufficiently.

4.3.4 Check on the Interpretations of the Step Test with the Pumping Well Drawdowns Observed During the Constant-Rate Pumping Test

After the full-scale constant-rate pumping test has started, it is useful to confirm that the results of the pumping test are consistent with the step test that preceded it. This confirmation of the consistency can provide an early check on whether it is safe to extrapolate the results of the step tests to longer durations of pumping.

A constant-rate pumping test was conducted with the same pumping well after the step test introduced in Section 4.3.1 was completed. The complete drawdown record for the pumping well is plotted in Figure 4-9. It is clear from the plot that the major portion of the drawdown is established almost immediately after the start of the test. This confirms that the additional well losses are significant.

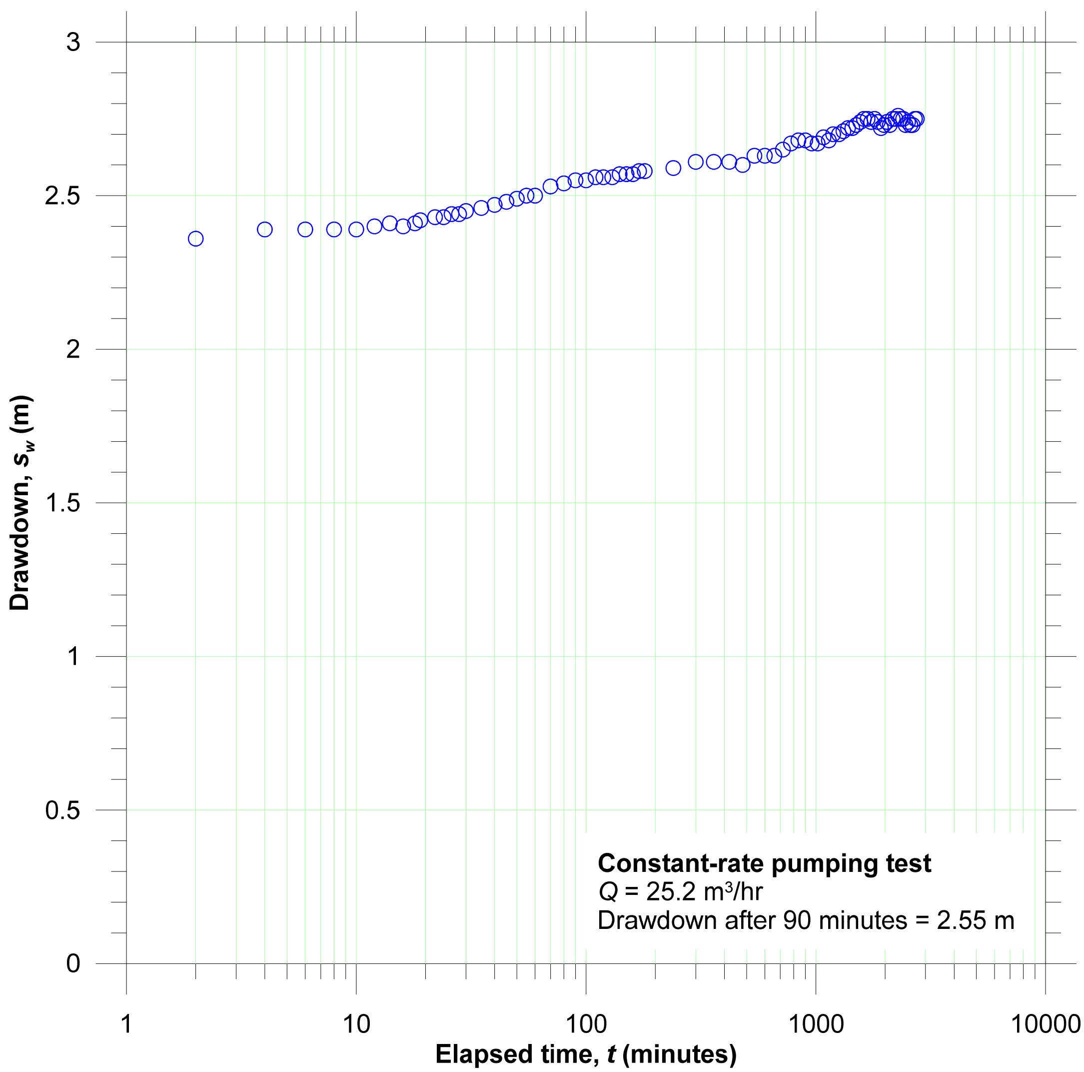

Figure 4-9. Pumping well drawdowns during the constant-rate test.

The constant-rate pumping test was conducted at a rate of 25.2 m^3/hr for 2760 minutes (46 hours). After 90 minutes, which corresponds to the duration of each pumping interval during the step test, the drawdown was 2.55 m. This additional point is added to the Hantush-Bierschenk plot in Figure 4-10. As shown in the figure, the results from the constant-rate pumping test are consistent with the results of the step test. It is recommended that this check be conducted during the pumping test, while there is still the opportunity to adjust the pumping rate.

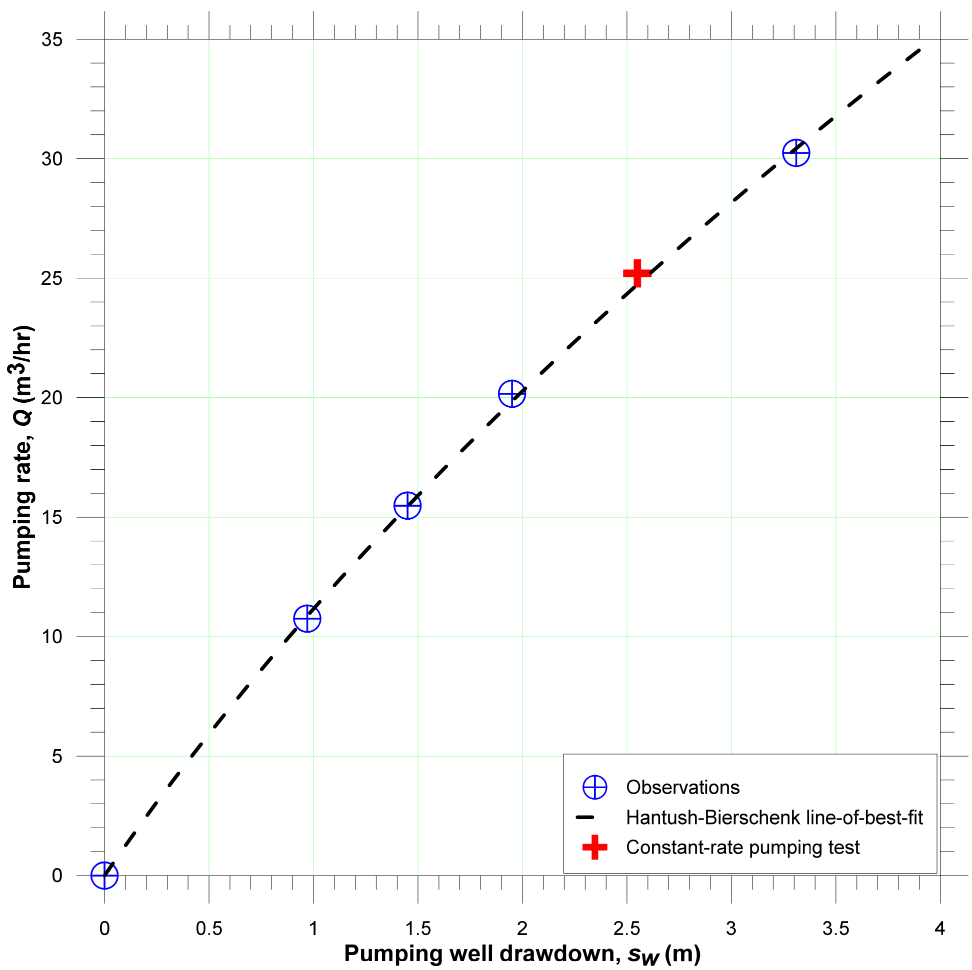

Figure 4-10. Check on the consistency of the step and constant-rate pumping tests.

4.3.5 Estimation of the Transmissivity from the Pumping Well Drawdowns During the Constant-Rate Pumping Test

A complete analysis of the response to pumping involves inferring the conceptual model that is consistent with the observed responses and estimating the values of the parameters for that model. We refer to this as a *comprehensive analysis*. We have recommended that prior to moving on to a comprehensive analysis, simple analyses be conducted to develop preliminary estimates of aquifer properties that can be used as a "reality check" for more refined calculations. Continuing with this approach, we recommend that a Cooper-Jacob straight-line analysis be conducted with the drawdowns observed in the pumping well during a constant-rate pumping test.

In addition to being simple to execute, the Cooper-Jacob analysis has the strength of "filtering" complications that may affect the interpretation of pumping tests with more complex conceptual models. These complications include skin losses, losses arising from partial penetration and nonlinear well losses, and the complexities introduced by small-scale heterogeneities. Analyses with more complex conceptual models are affected by these complications, as the observed total drawdown is matched. In contrast, the Cooper-Jacob analysis matches the rate of change of drawdown, which is controlled only by the bulk average transmissivity of the formation (Butler, 1990; Meier et al. 1998). The simplicity of the Cooper-Jacob analysis is in some ways deceptive, as it frequently yields the most representative estimate of the transmissivity.

An example illustration of the Cooper-Jacob straight-line analysis is shown in Figure 4-11. The data were collected during a constant-rate pumping test conducted in a buried-valley aquifer near Estevan, Saskatchewan. The test will be discussed in more detail in Appendix 4.

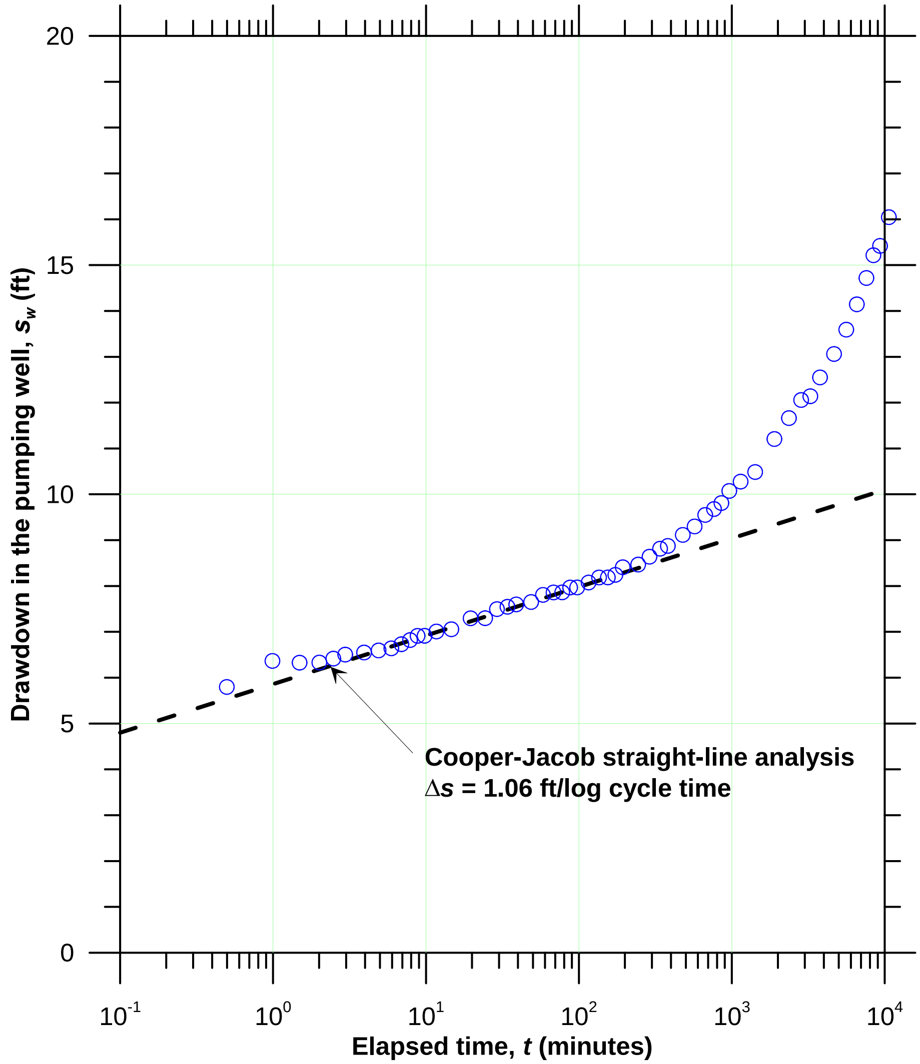

Figure 4-11. Estimation of transmissivity with Cooper-Jacob straight-line analysis.

The average pumping rate during the test was 460 (Igpm) [3,010 m^3/d]. The slope of the straight-line portion of the response, the interval that is assumed to be representative of the ideal aquifer response, is 1.06 ft/log cycle time [0.323 m/log cycle]. The transmissivity is therefore estimated as:

$$T = 2.303 \frac{Q}{4\pi} \frac{1}{SLOPE}$$

$$= 2.303 \frac{(460\ Igpm)}{4\pi} \frac{1}{(1.06\ ft)} \left| \frac{m^3}{219.97\ Igal} \right| \left| \frac{1440\ min}{d} \right| \left| \frac{3.281\ ft}{m} \right|$$

$$= 1{,}708 \frac{m^2}{d};\ \text{That is, about } 1{,}700\ \mathrm{m^2/d}$$

The key assumption of the Cooper-Jacob analysis is that the interval of the response on the semi-log plot that is fit with a straight line is indeed representative of ideal conditions. More precisely, the interval that is matched must correspond to the period during which there is bulk-average radial flow to the pumping well. Inspection of Figure 4-11 suggests that two straight lines can be constructed through the drawdown data, between about 1 minute and 200 minutes, and between about 4,000 minutes and 10,000 minutes. The execution of an appropriate Cooper-Jacob analysis can be enhanced with derivative analysis. Computer-assisted methods of interpretation can be used to plot simultaneously the drawdown data and the smoothed derivative of the drawdown, as shown in Figure 4-12. The Cooper-Jacob straight-line should be fit over the interval of time during which the smoothed derivative reaches a plateau. This plateau is designated the period of *Infinite Acting Radial Flow*, *IARF*.

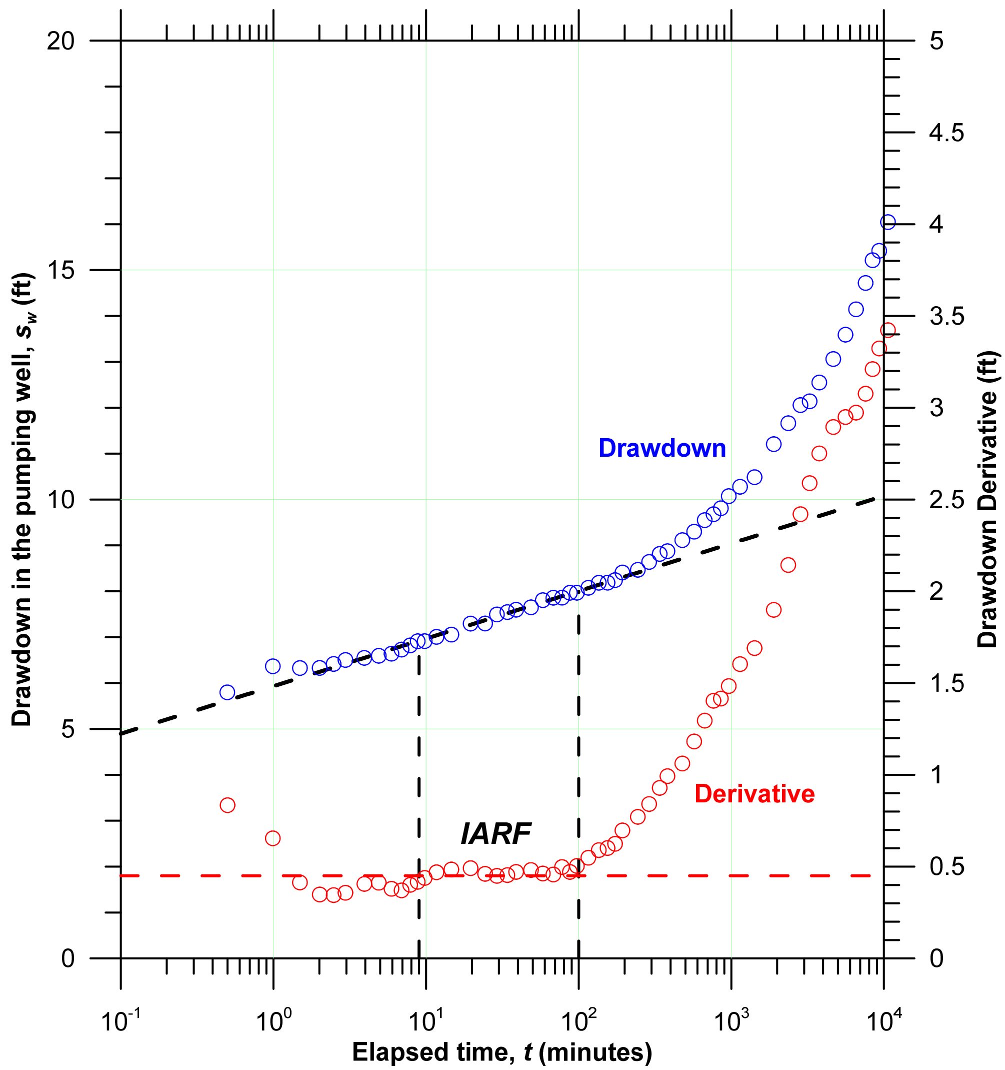

Figure 4-12. Estimation of transmissivity with Cooper-Jacob straight-line analysis, supplemented with derivative plot.

The Cooper-Jacob analysis also yields an estimate of the confined storage coefficient, S (also referred to as the storativity). The storage coefficient it is not used directly in the either the estimation of the transmissivity or the safe yield; however, its estimation has important diagnostic value. The storage coefficient estimated for the Estevan pumping test is **4.3×10^{-4}**. This value is within the range of typical values for confined sand and gravel aquifers (Lohman, 1972; Boonstra, 1989). This suggests that additional well losses may not be significant in this case. Although this assessment is useful, it is certainly not definitive. More definitive interpretations should be based on the results of step tests.

4.3.6 Estimation of Transmissivity from the Drawdowns During the Period Of "Ideal" Aquifer Response

When drawdown data are available for the pumping well and observation wells, we recommend that a single analysis be developed to analyze all of the available data. It is important to note that all "conventional" methods of analyses, that is, methods that are based on analytical solutions, incorporate the fundamental assumption that the aquifer is homogeneous. This implies that for an analysis to be valid, consistent parameters must be estimated for the pumping well and the individual observation wells that are located in the pumped aquifer. Estimation of an inconsistent set of parameters indicates only that the major assumption underlying the analysis has been violated, such that the analysis may not be valid. This may point to the need to invoke a conceptual model that goes beyond the Theis model.

Cooper and Jacob (1946; p. 529) indicated that when the drawdown data are available for several observation wells at different times, the drawdowns should be plotted against the logarithm of t/r^2, where t is the elapsed time and r is the distance between the pumping well and each observation well. This is referred to as a *composite plot*. Weeks (1977) writes, "The composite data-curve matching process is also important during the analysis of test data. Such data should always be made when data from more than one observation well are available."

The data from a pumping test conducted in a limestone and dolostone aquifer are presented to illustrate an analysis with a composite plot. The drawdown data from the individual wells are plotted in Figure 4-13. The corresponding composite plot is shown in Figure 4-14. For an aquifer that conforms to the assumptions of the Theis aquifer, beyond the initial period of response all of the drawdown data should approximate a single line. The fact that the data do not approximate a single line in Figure 4-14 confirms that the aquifer is not ideal.

The analyses of Butler (1990) and Meier and others (1998) have shown that the Cooper-Jacob analysis is not invalidated when the data approximate multiple parallel lines as they do in Figure 4-14. In this case, the slope of the parallel lines yields a consistent estimate of the bulk-average transmissivity of the formation. Each straight line yields a different estimate of the storage coefficient. The individual estimates of the storage coefficient are not reliable; the different estimates are diagnostic of a heterogeneous formation.

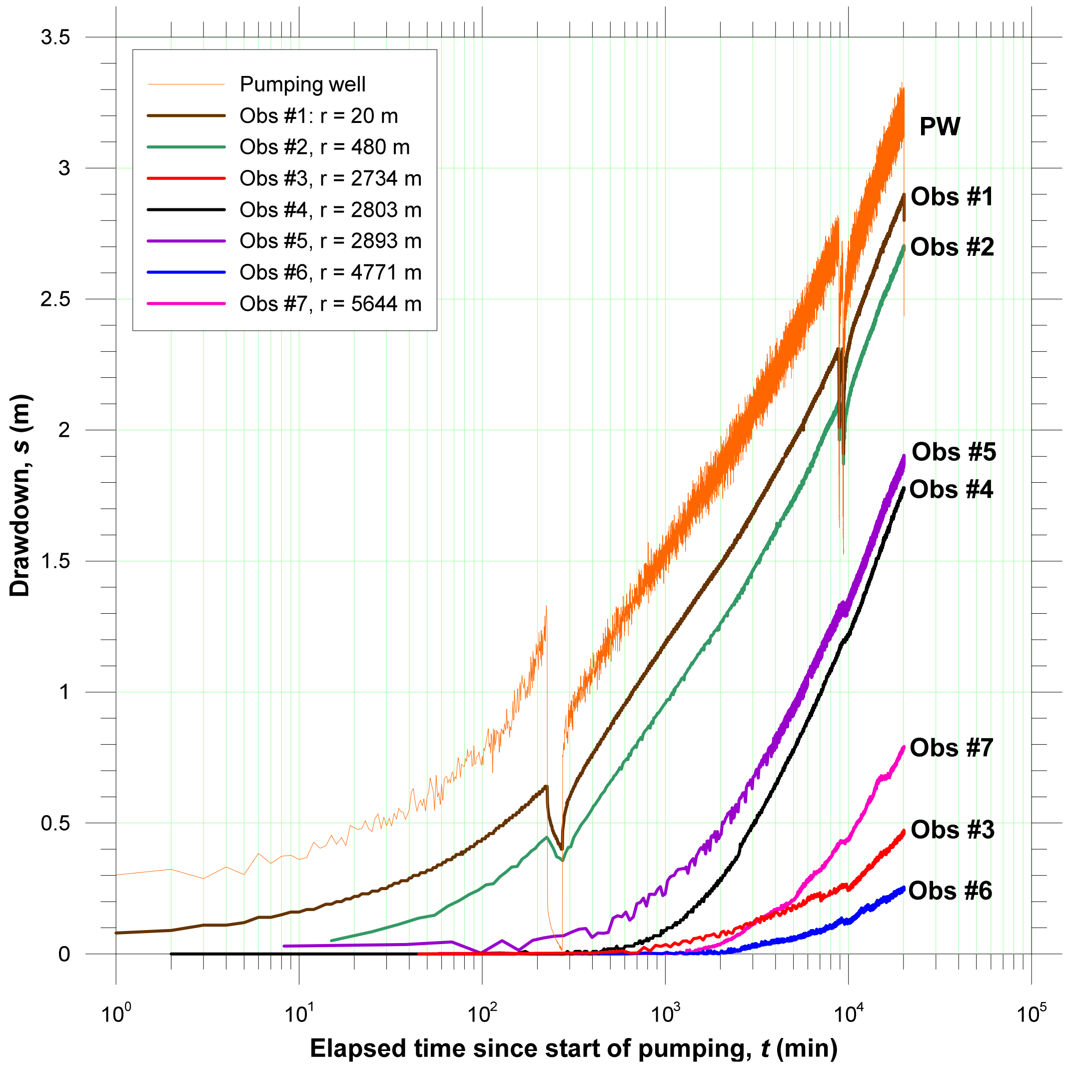

Figure 4-13. Drawdown records for the pumping well and multiple observation wells.

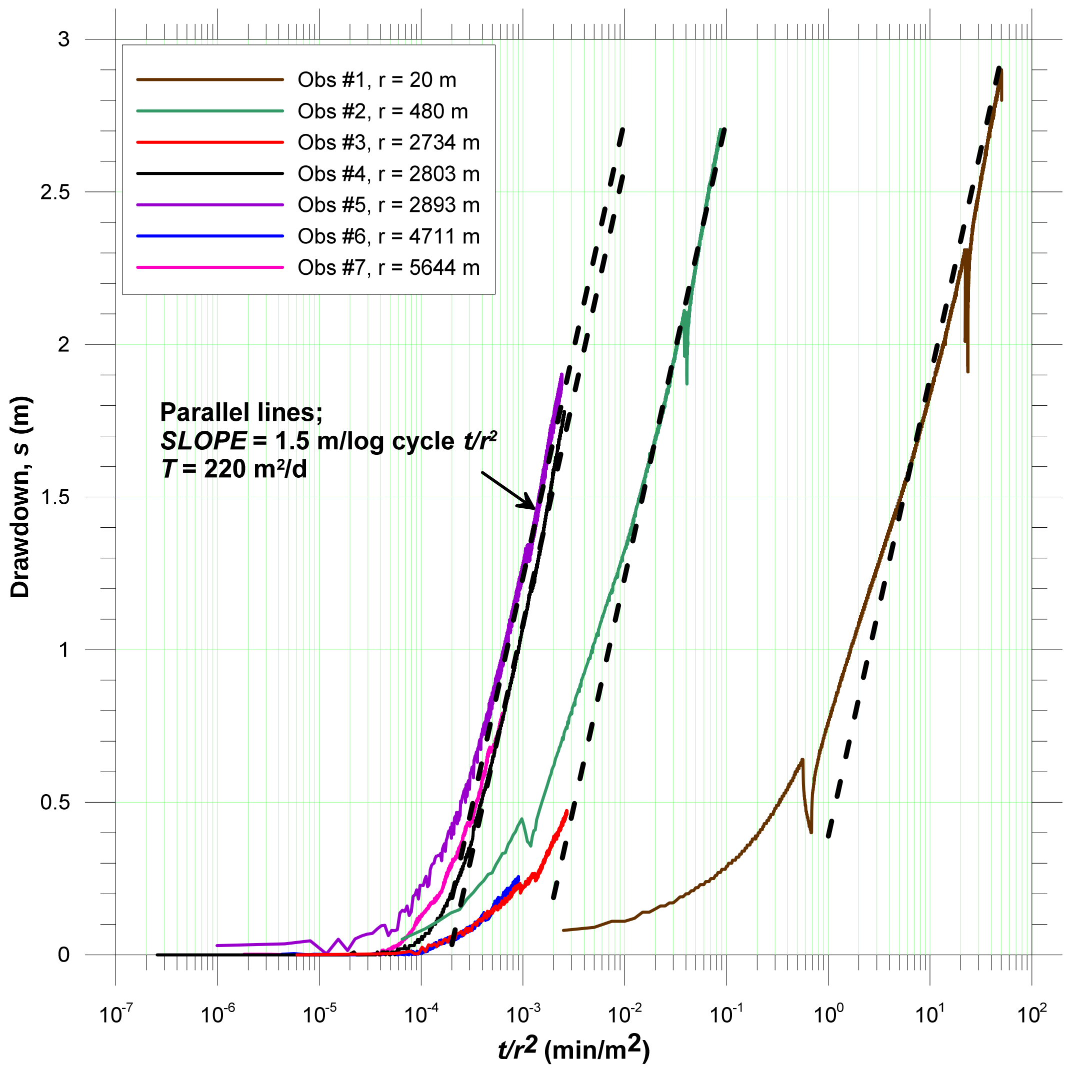

Figure 4-14. Composite plot of observation well drawdowns, with Cooper-Jacob analysis.

If the only purpose of conducting a pumping test is to estimate the bulk-average transmissivity, the Cooper-Jacob straight-line analysis with a composite plot generally yields the most representative estimate, provided the straight-line analysis is conducted over the appropriate range of values of time or t/r^2 (Butler, 1990; Meier and others, 1998). There is generally some portion of the response during which there is mean-radial flow directed towards the pumping well. In the case of an aquifer that is confined between compressible aquitard units across which there is significant leakage, the period of mean-radial flow may be relatively early during a pumping test. In the case of an unconfined aquifer, the period of mean radial-flow may be established only after a relatively long time. Drawdowns observed during a long-term pumping test in an unconfined aquifer are shown in Figure 4-156. As shown in the figure, it is possible to match the data with a consistent set of parameters; in this case, the fit was obtained after significant effort with a least-squares matching optimization algorithm with the relatively complex Neuman (1974) solution.

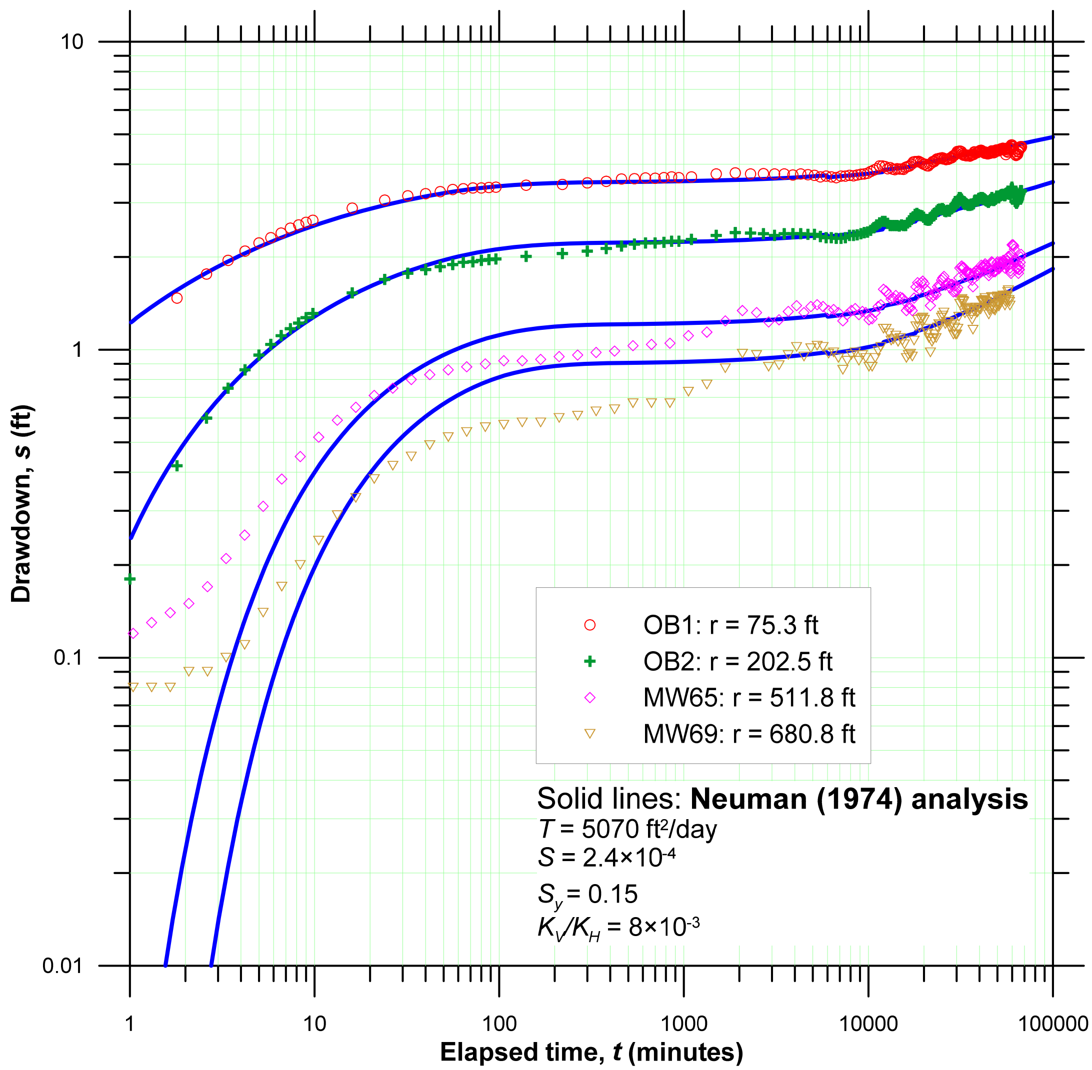

Figure 4-15. Pumping test in an unconfined aquifer.

In Figure 4-16, the drawdown data from Figure 4-15 are assembled on a composite plot with semilog axes. Only a straightedge and calculator are required to obtain an estimate of the transmissivity that is consistent with the data from all four wells, for both the early-time (confined) and the late-time (unconfined) portions of the response. The Cooper-Jacob analysis is clearly simpler; however, that is not the prime motivation for using this approach. The composite plot aids in synthesizing the data. The different periods of the response are distinguished more clearly on a semilog plot; the transition period between the "pure" confined and "unconfined" responses is particularly clear. The visualization of the drawdown data is further enhanced with the addition of the derivative, as shown in Figure 4-17.

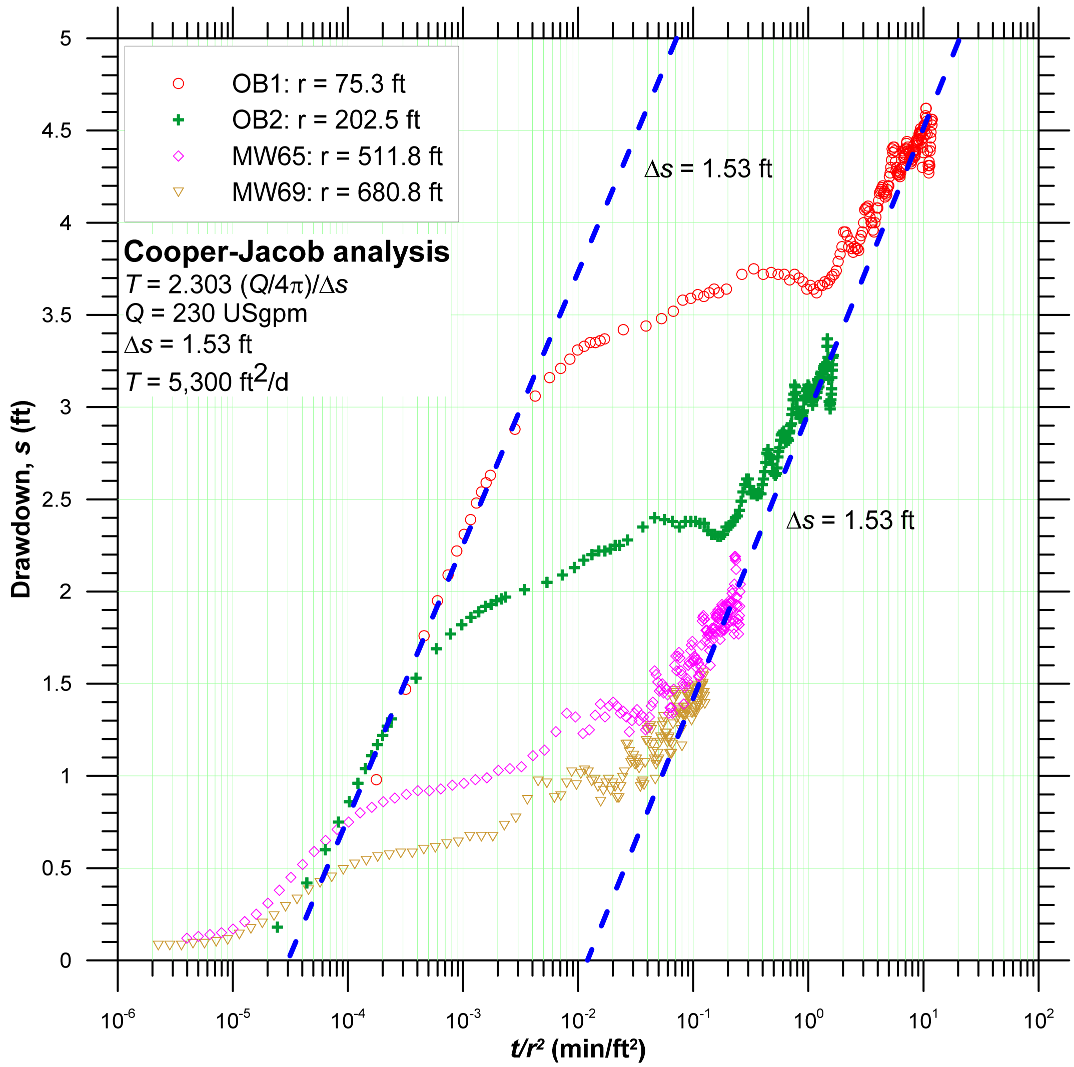

Figure 4-16. Pumping test in an unconfined aquifer, Cooper-Jacob composite analysis.

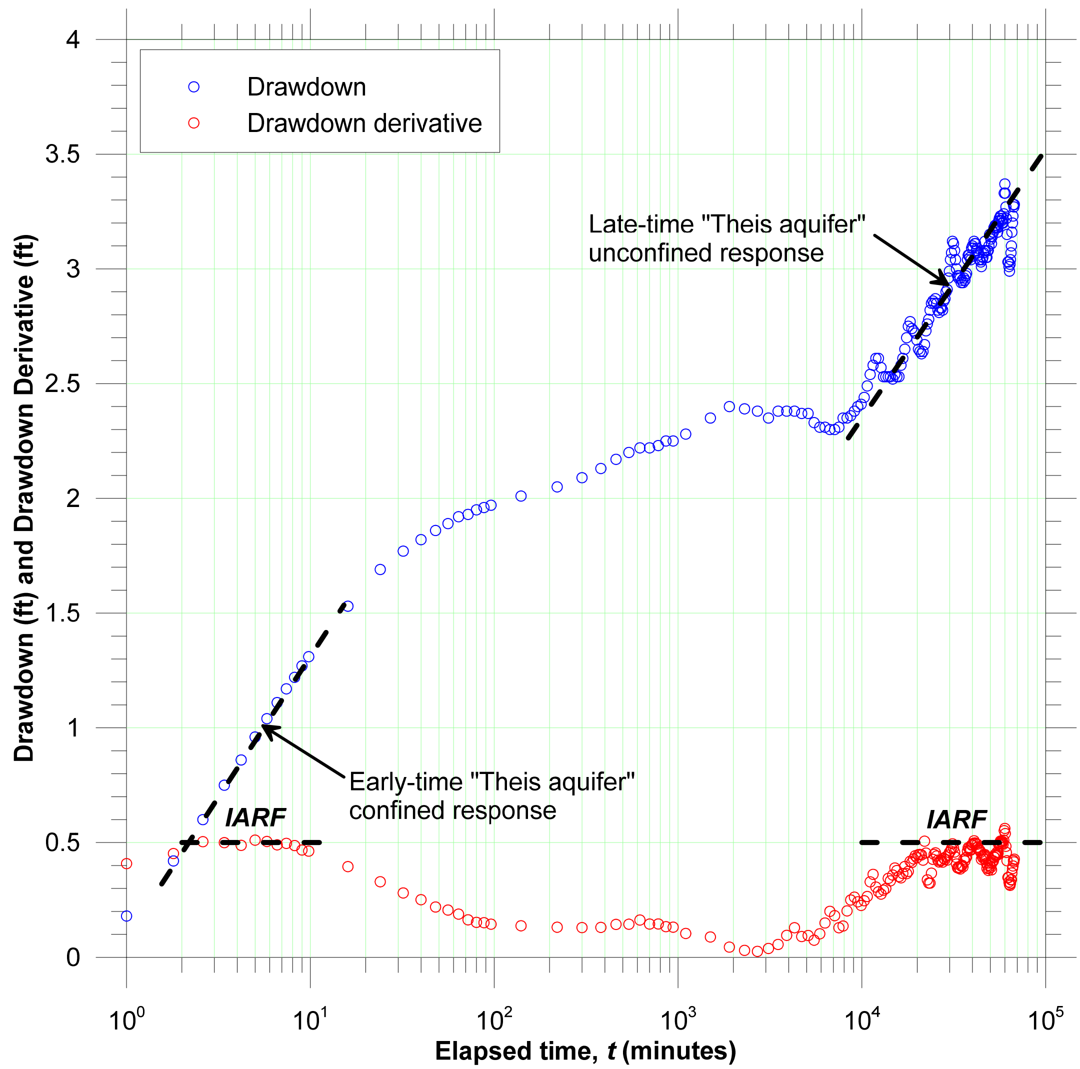

Figure 4-17. Pumping well drawdown and drawdown derivative.

4.3.7 Diagnosis Of The Complete Response To Pumping

The estimation of the bulk-average transmissivity is generally only one of the objectives for conducting a pumping test. The most important reason to conduct a pumping test is more general: to observe how the groundwater system responds to a controlled stress, and thereby gain some insight into the structure of the system. Although we stand by our recommendation to begin the interpretation by using simple conceptual models to estimate the transmissivity of an aquifer, the estimation of the transmissivity is generally not sufficient for a complete interpretation. If the interpretations of a pumping test are to guide any future decision-making, , interpretation of the pumping test should confirm that the aquifer is sufficiently well understood. This confirmation requires that representative and non-representative responses to pumping be identified, and that the complete set of representative responses to pumping be matched with an appropriate conceptual model with a consistent set of parameters.

Inferring the appropriate conceptual model and distinguishing between representative and non-representative responses is rarely clear cut. Inference of the appropriate conceptual model requires an appreciation of the geologic setting, an aversion to preconceived notions, and a keen eye. It is also important to bear in mind that, although the number of analytical solutions available for the interpretation of pumping tests is large, these solutions have all been developed for highly idealized settings. There is a growing recognition that, in many instances, a complete interpretation may require the development of a numerical model (see for example, Schroth and Narasimhan, 1997; Johnson et al., 2002; Spiliotopoulos and Andrews, 2006).

To provide some sense of the approaches that can be adopted for a complete analysis based on conceptual models that are still tractable for analytical solutions, the analysis of the pumping test introduced in Section 4.3.6 is revisited. A complete case study of a complete analysis of a pumping test conducted in a buried-valley aquifer is presented in Section 6.

Recommended methods for the estimation of the transmissivity presented in this document have relied heavily on the application of Cooper-Jacob straight-line methods. It is important to note that the diagnosis of the conceptual model requires several alternative forms of plots, including plots with bi-logarithmic axes, and plots of the drawdown derivative with semilog and log-log axes. Responses for a selection of idealized conceptual models are compiled in Appendix 1. Ehlig-Economides (1988) and Renard and others (2009) have assembled very useful collections of drawdown and derivative patterns.

Example analysis

Drawdown data from a pumping test were presented in Figure 4-13, and an estimation of the transmissivity was presented in Figure 4-14. A consistent estimate of the bulk-average transmissivity was obtained, but the deviation of the response from the ideal indicated clearly that the aquifer was complex. To supplement the analysis, the drawdowns at the end of pumping are plotted against the distance from the pumping well in Figure 4-18. In an ideal confined aquifer, the drawdowns should lie on a single straight line when plotted on semi-log axes. The departure from a single straight line is further confirmation that the aquifer is heterogeneous.

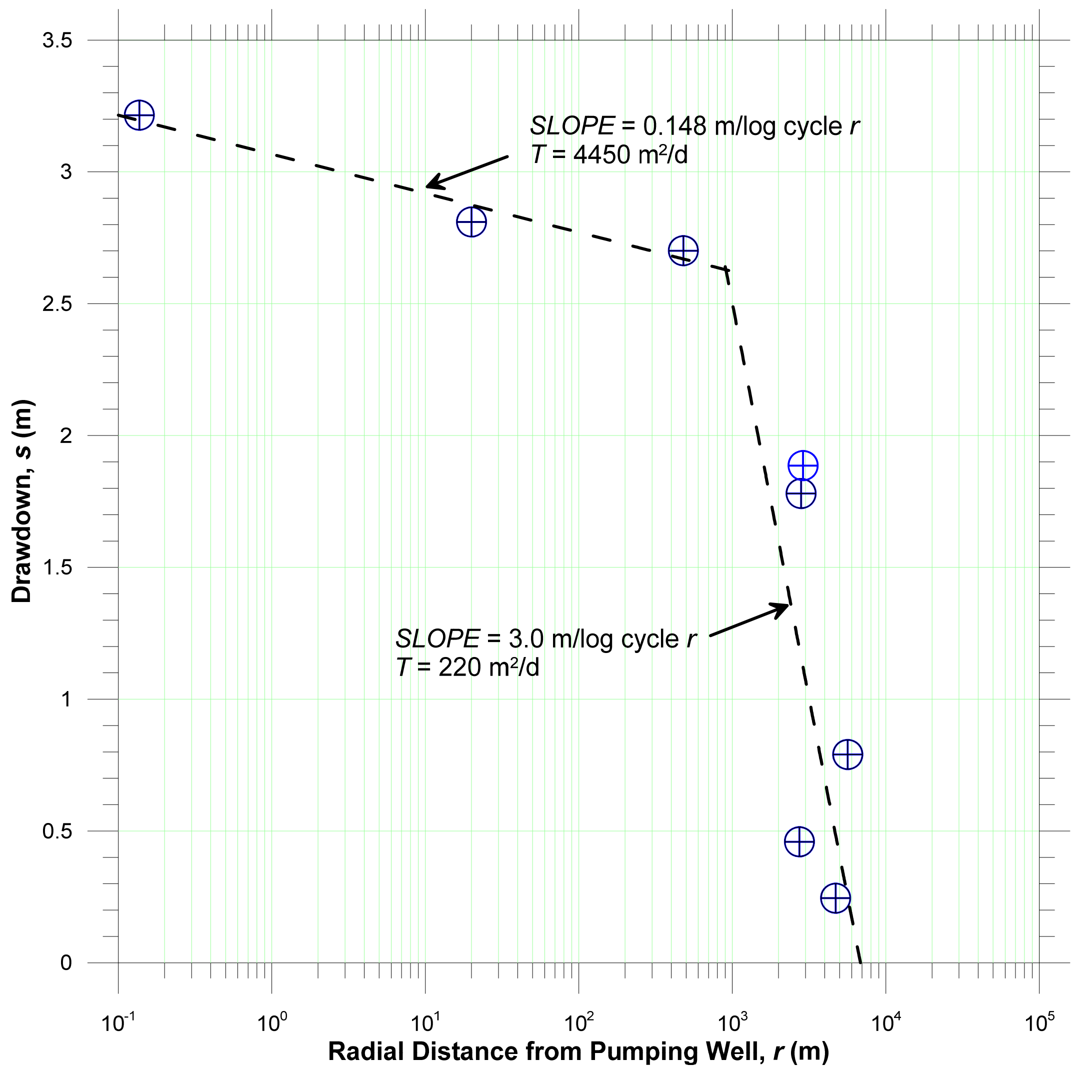

Figure 4-18. Distance-drawdown plot.

As shown in Figure 4-18, the drawdowns appear to approximate two straight lines, with the wells between 0.1 m and 1,000 m approximating one line and the wells between 1,000 m and 10,000 m approximating another. A separate Cooper-Jacob straight-line analysis can be conducted for each straight line in Figure 4-18:

Group 1: 0.1 m < *r* < 1,000 m

$$T_1 = 2.303 \frac{Q}{2\pi} \frac{1}{\Delta s_1}$$

The slope for the wells in the first group is about 0.15 m per log cycle of distance. For a pumping rate of 1,800 m^3/d, this yields a transmissivity of **$\underline{4{,}450\ m^2/d}$**.

Group 2: 1,000 m < *r* < 10,000 m

$$T_2 = 2.303 \frac{Q}{4\pi} \frac{1}{\Delta s_2}$$

The slope for the wells in the second group is about 3.0 m per log cycle of distance. This yields a transmissivity of **$\underline{220\ m^2/d}$**.

The results of the distance-drawdown analysis for wells beyond 1,000 m from the pumping well are consistent with the results of the composite analysis presented in Figure 4-14. The distance-drawdown plot suggests that the aquifer responds as if the pumping well is surrounded by a zone that has elevated transmissivity with respect to the surrounding formation. To test this hypothesis, the pumping test is simulated with an analytical solution that implements a conceptual model of radial heterogeneity (Barker and Herbert, 1982). As shown schematically in Figure 4-19, the assumptions of the Theis model are retained in the Barker-Herbert model, with the exception of the assumption of homogeneity. The aquifer is assumed to consist of a zone with one set of properties surrounding the pumping well, surrounded by a zone of uniform properties corresponding to the bulk formation.

Inner zone: $r < R$, $T = T_1$, $S = S_1$
Outer zone (formation): $r > R$, $T = T_2$, $S = S_2$

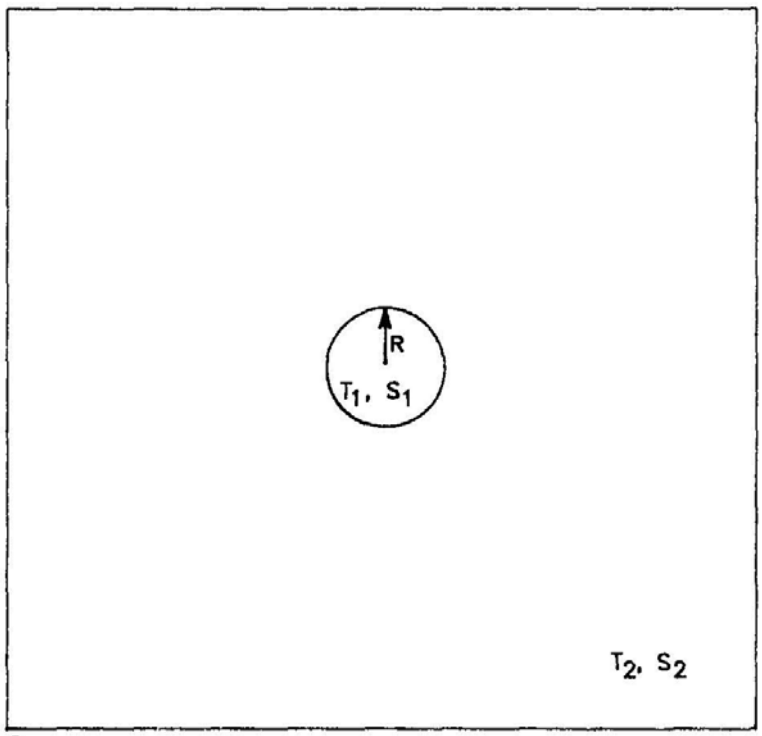

Figure 4-19. Barker and Herbert (1982) conceptual model.

The transmissivity values obtained from the Cooper-Jacob distance-drawdown analysis are specified in the Barker-Herbert solution (T_1 = 4450 m^2/d; T_2 = 220 m^2/d). To simplify the analysis, it is assumed that the storage coefficients S_1 and S_2 are both 1.5×10^{-5}. Referring again to Figure 4-18, the zone of elevated transmissivity is assumed to surround the pumping well to a distance of 1,000 m. The results of the Barker-Herbert solution are plotted in Figure 4-20. As shown in the figure, an excellent match is obtained to the final drawdowns.

The close match to the observations supports the hypothesis that the pumping well is surrounded by a zone that has elevated transmissivity with respect to the surrounding formation.

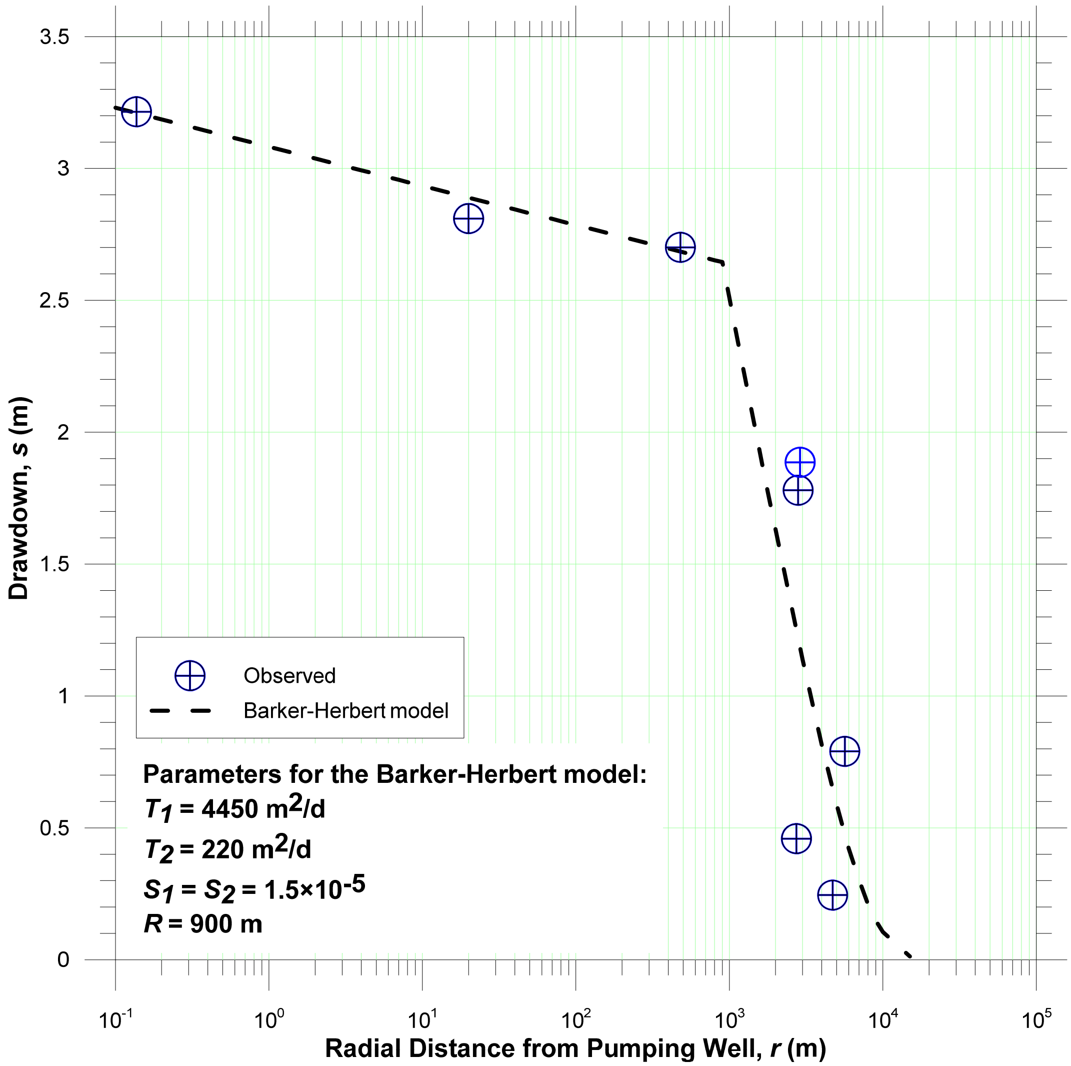

Figure 4-20. Distance-drawdown plot with results of the Barker-Herbert model.

4.4 Decision Analysis For The Interpretation Of Pumping Tests

Assumption of the Theis model	**Dealing with departures from the ideal model**
There are no variations in water levels during the pumping test that are not due to pumping.	Before any pumping test analysis is conducted, the water level data must be processed to remove any changes in water levels that are not caused by pumping.
Darcy's law is valid.	In sand and gravel aquifers, groundwater velocities will be within the limits of validity of Darcy's Law if the well has been designed properly. In fractured-rock aquifers in which flow is limited to discrete features, flow close to the well may be turbulent and give rise to drawdowns in excess of those that would be observed if Darcy's Law is valid. Violation of Darcy's Law will be exhibited as elevated nonlinear well losses during a step test.
The aquifer is horizontal.	Pumping tests in aquifers that are not horizontal can be analyzed with methods for horizontal aquifers. For steeply dipping aquifers, it is important to identify those observation wells that are in the same aquifer as the pumping well. The distances between wells should be measured in the plane of the aquifer, and not with respect to the coordinates at ground surface.
The aquifer has a constant thickness.	An analytical solution exists for ideal wedge-shaped aquifers [Hantush, 1962]. If the aquifer has a complex structure and its thickness varies significantly, it may be necessary to develop a numerical model to interpret the pumping test.
The aquifer is infinite in areal extent.	The principle of superposition (image wells) can be used to incorporate linear zero-drawdown and no-flow boundaries of infinite extent. If the aquifer has a complex structure and boundary conditions that are relatively close to the pumping well, it may be necessary to develop a numerical model to interpret the pumping test.

Decision analysis for the interpretation of pumping tests, continued

Assumption of the Theis model	Dealing with departures from the ideal model
The hydraulic conductivity is uniform.	When interpreted appropriately, a pumping test yields the bulk-average transmissivity of the aquifer. A pumping test does **not** yield reliable estimates of the properties of small zones within the drawdown cone. Analytical solutions exist for some simple models of aquifer heterogeneity. If the aquifer has a complex structure and there are large zones with different properties, it may be necessary to develop a numerical model to interpret the pumping test.
The hydraulic conductivity is isotropic.	An analytical solution exists for a simple model of anisotropy in the horizontal plane [Papadopulos, 1965]. Anisotropy in the vertical plane can only be detected with partially penetrating pumping wells and observation wells that are relatively close to the pumping well [see "The pumping well penetrates the full thickness of the aquifer."]
The aquifer is perfectly confined along its top and bottom by impermeable strata.	There are several analytical solutions of varying complexity for pumping tests in which there is significant leakage from confining units. These solutions all assume that the aquitards are significantly less permeable than the pumped aquifer [Hantush and Jacob (1955), Hantush (1960), Neuman and Witherspoon (1969), Cooley and Case (1973), Moench (1985)]. A numerical model must be developed to analyze the complete response during a pumping test in a system where the contrasts in the hydraulic conductivities of different strata are not large.
The piezometric surface in the pumped aquifer always remains above the top of the aquifer.	If the piezometric surface does not remain above the top of the aquifer, the source of water for the pumping well changes from the release of water from confined storage to the drainage of the pores at the water table. An analytical solution exists for a pumping test in which pumping causes a conversion from confined to unconfined conditions (that is, the water level in the aquifer is brought below the top of the aquifer)[Moench and Prickett, 1972]. There are several analytical solutions of varying complexity for pumping tests in aquifers that remain unconfined during the duration of a pumping test [e.g., Neuman (1972), Neuman (1974), Moench, 1997, Tartakovsky and Neuman, 2007].

5. ESTIMATION OF THE LONG-TERM YIELD OF A WELL

Overview

Estimation of the long-term yield of a pumping well is one of most important analyses a hydrogeologist can undertake. Historically, the long-term yield of a well in Alberta has been estimated using the Q_{20} approach (Farvolden, 1959; Bibby, 1979; Weyer, 2003). The updated *Guide to Groundwater Authorization* (Alberta Environment, 2011) recommends that the Q_{20} calculation be replaced with the Modified Moell method (Maathuis and van der Kamp, 2006). Application of the Q_{20} and Modified Moell methods are illustrated through the case study presented in Appendix 4. The results of the analyses for this case study demonstrate that it is crucial to constrain the extrapolation by invoking a representative conceptual model for the aquifer.

5.1 Definitions Of Safe Yield And Sustainable Yield

We begin by making an important distinction between two concepts that have been associated with the long-term yield of wells: the *safe yield of a well* and the *sustainable yield of a well*.

Safe yield of a well: The safe yield of a well is defined here as the average rate at which a well can be pumped without the water level in the well declining below a minimum level after a specified duration of pumping. That minimum level might be a few metres above the top of a confined aquifer, or several metres above the screened section of a well in an unconfined aquifer. The duration of pumping may be days, months, or years, depending on the objectives of pumping.

Sustainable yield of a well: The sustainable yield of a well is defined here as the average rate at which a well can be pumped without there being unacceptable impacts after a specified duration of pumping.

The definition of the sustainable yield offered here is related closely to, but is slightly more specific than, the definition of safe yield proposed by Todd (1959): "the amount of water that can be withdrawn annually without producing an undesired result." The present definition of sustainable yield is consistent with an inclusive definition of sustainable water resource developments proposed by the American Society of Civil Engineers as those developments that are designed and managed "to maintain ecological, environmental, and hydrological integrity" (ASCE, 1998).

In the remainder of this section, attention is limited to the estimation of the safe yield of a well, recognizing that, in general, an assessment is not complete if it assumes that the only potentially undesirable effect of pumping is the lowering of the water level in the pumping well beneath a specified minimum level.

5.2 The Role Of Step Tests In Establishing The Safe Yield Of A Well

Every pumping test should be preceded by a step test. A step test is a test during which the production well is pumped at a sequence of increasing rates, or steps, of relatively brief and equal duration (typically 30 minutes to 1 hour). A step test is conducted to establish the rate at which a well can be pumped for a test lasting several days and to provide insights regarding the performance of the pumping well.

If a step test is not conducted prior to a constant-rate pumping test, there can be no assurance that the well can be pumped at the planned rate for the duration of the constant-rate test. The importance of a step test is illustrated with the results of a step test conducted in a dolostone aquifer shown in Figure 5-1. Based on the results of previous slug and packer tests, a seven-day pumping test was planned to be conducted at a constant rate of 20 L/s. The step test was designed to be conducted with five steps at increments of 5 L/s, each lasting 30 minutes. As shown in Figure 5-1, the water level in the pumped well declined precipitously when the pumping rate increased from 10 L/s to 15 L/s. In this case, the unexpected response was not due to an underestimation of the transmissivity of the aquifer at the location of the well. Rather, it was found subsequently that the well derived its supply from a major water-producing zone at an elevation of about 263 m ASL.

Following the analysis of the results of the step test, it was decided to conduct the constant-rate pumping test at a rate of 10 L/s. As shown in Figure 5-2, the water level in the pumping well remained above the level of the major water-producing zone during the entire duration of the seven-day constant-rate test.

Although seven days is relatively long for a constant-rate pumping test, it is brief compared to the lifespan of a typical well installed to control groundwater inflows to an open-pit excavation, depressurize a confined aquifer beneath an excavation, or to provide a long-term water supply. We do not recommend that the results of a step test be used to estimate the long-term yield of a well.

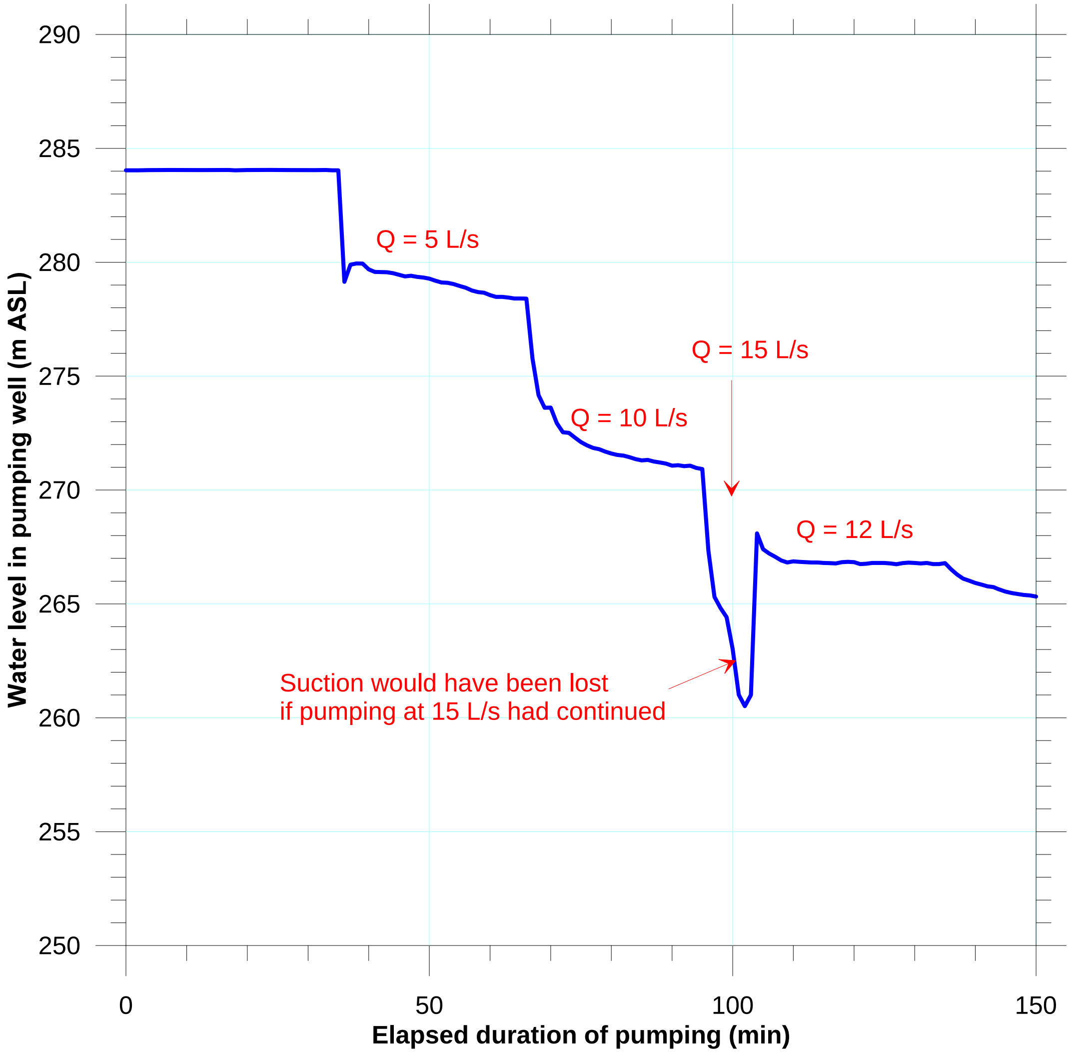

Figure 5-1. Results of step test conducted in a dolostone aquifer.

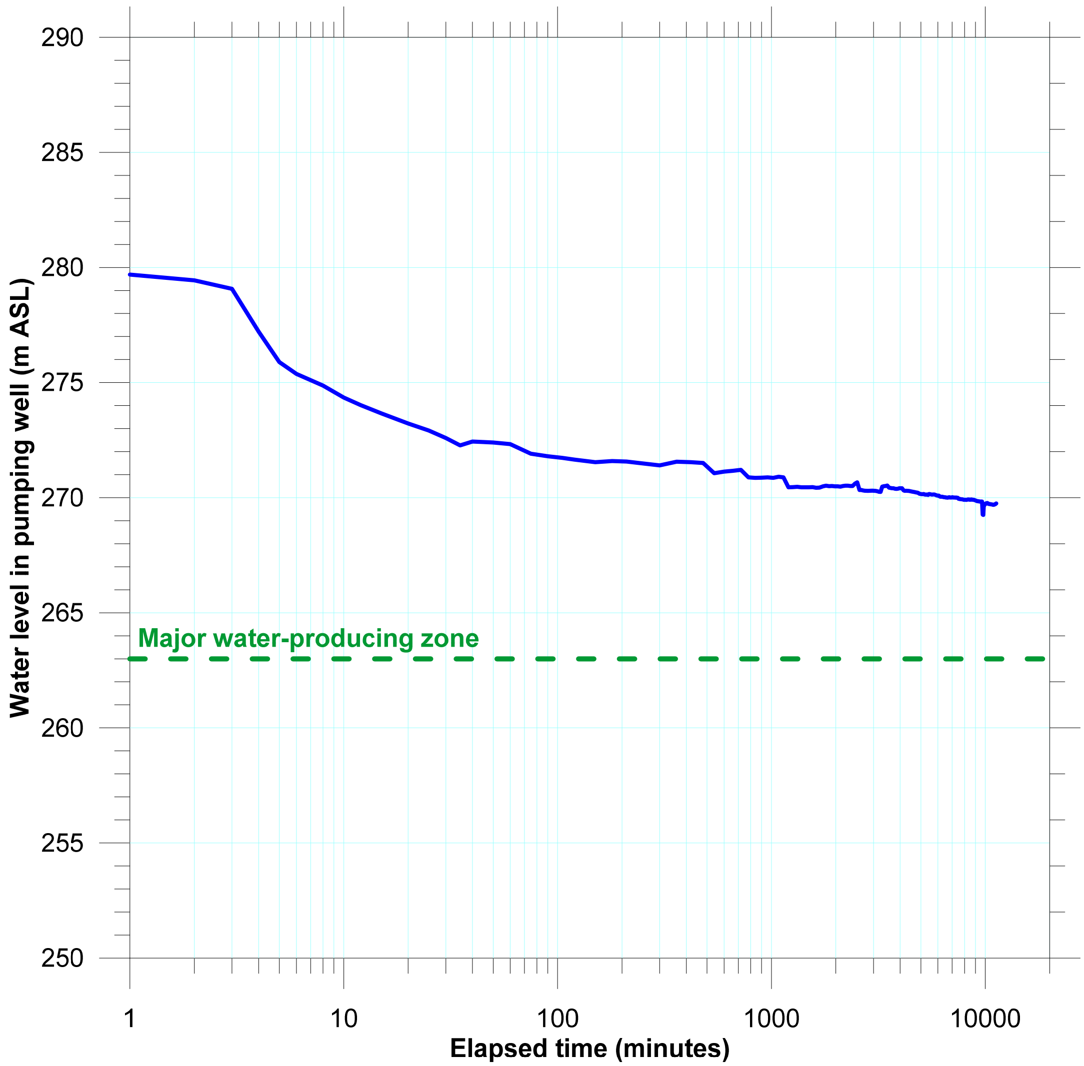

Figure 5-2. Results of constant-rate pumping test conducted in a dolostone aquifer.

5.3 Foundations And Assessment Of The Q_{20} Method

Regardless of the specific approach that is adopted to estimate the long-term yield of a well, it is important to recognize that there are many ways to be wrong. The estimation of the long-term yield involves extrapolation, an inherently risky undertaking.

The Q_{20} method is developed in Farvolden (1959). For a confined aquifer, the theoretical safe yield, Q_{20}, represents the pumping rate that may be supported for 20 years, without exceeding the available drawdown, H_A in an aquifer that has a transmissivity T. The available drawdown is interpreted as the difference between the water level in the absence of pumping and the elevation of the top of the aquifer. The safe yield is calculated according to:

$$Q_{20} = 0.7 \times 0.68\, T\, H_A \tag{5-1}$$

No details on the development of Equation (5-1) are presented in Farvolden (1959). To assess the Q_{20} method, it is first necessary to understand its foundations. The Q_{20} method is a direct application of the Cooper-Jacob approximation (Cooper and Jacob, 1946). The drawdown in the pumping well, $s(r_w,t)$, predicted with the Cooper-Jacob approximation is:

$$s(r_w, t) = \frac{Q}{4\pi T} 2.303 \log\left(\frac{2.25Tt}{r_w^2 S}\right) \tag{5-2}$$

Here Q is the pumping rate, r_w is the radius of the pumping well, t is the elapsed duration of pumping, and S is the storage coefficient (storativity).

The difference between the drawdowns at any two times, t_1 and t_2, is given by:

$$s(r_w, t_2) - s(r_w, t_1) = \frac{Q}{4\pi T} 2.303 \log\left(\frac{t_2}{t_1}\right) \tag{5-3}$$

If t_2 is taken as 20 years (7,300 days) and it assumed that the drawdown is relatively small before an elapsed time of about 10 seconds, the ratio in the log term is about 10^8. Therefore, Equation (5-3) reduces to:

$$s(r_w,\ 20\ years) \approx \frac{Q}{4\pi T} 2.303\ (8) \tag{5-4}$$

Fixing the drawdown after 20 years as the allowable drawdown and solving for Q yields:

$$Q = \frac{1}{2.303\ (8)} 4\pi T\ H_A = 0.68\ T\ H_A \tag{5-5}$$

Finally, the Q_{20} value is calculated by applying a "factor of safety" to Equation (5-5), yielding Equation (5-1). The calculations are illustrated in Figure 5-3.

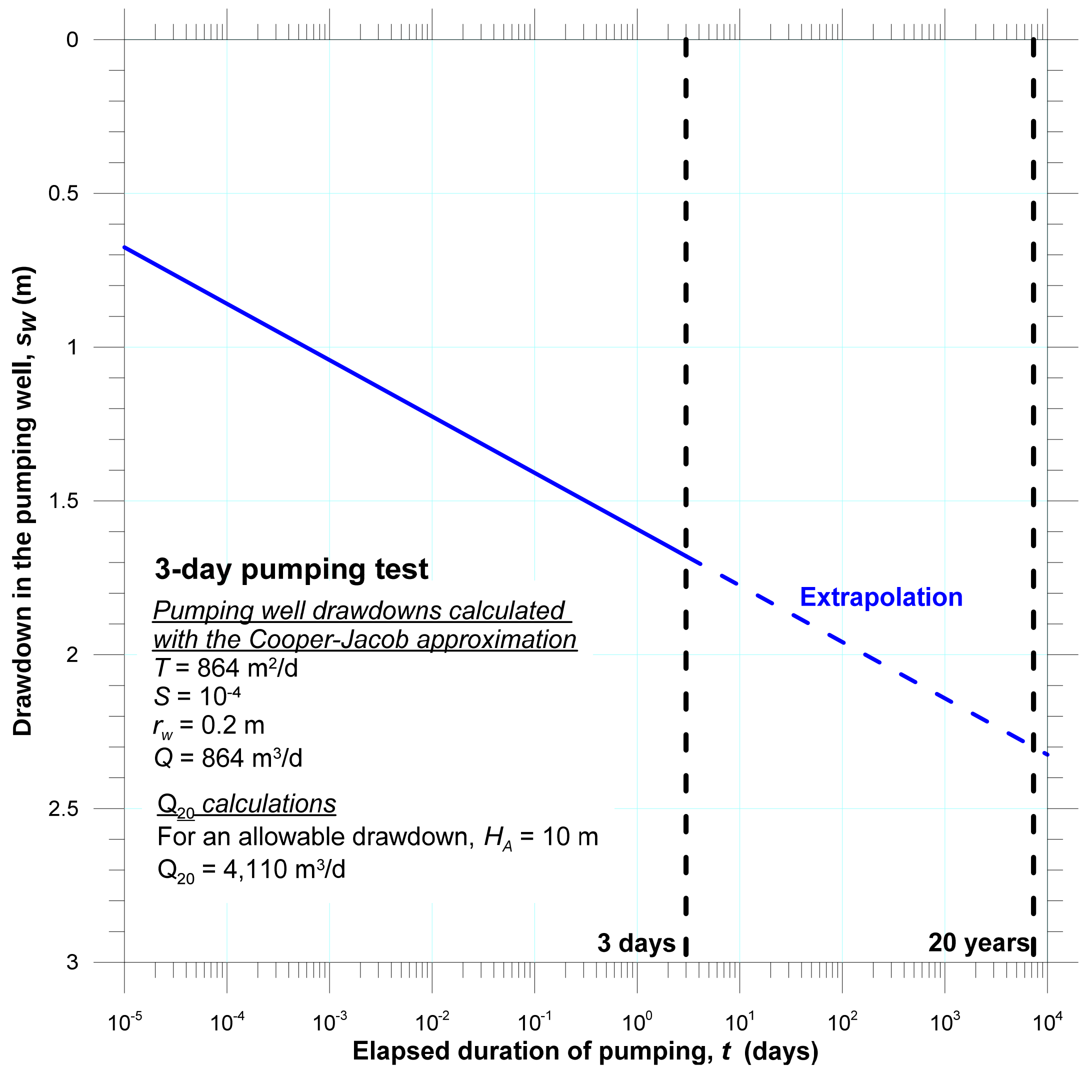

Figure 5-3. Example Q_{20} calculations.

The Cooper-Jacob approximation, and the Theis (1935) solution from which it is derived, are based on a highly idealized representation of the aquifer. Key assumptions of the conceptual model include:

- The well is perfectly efficient;
- The aquifer is of infinite areal extent;
- The aquifer is confined between completely impermeable strata; and
- The water level remains above the top of the aquifer at all times.

These assumptions imply that the drawdown in the pumping well is due only to head losses in the formation and that the only source of water for the pumped well is confined storage. The drawdown cone must therefore expand indefinitely in space and time.

A conceptual model that assumes that a production well is perfectly efficient is not realistic. Additional well losses will arise for several reasons. First, installation of the well will generally result in some alteration of the properties of the formation surrounding the well. Additional losses will occur if the well screen does not penetrate the full thickness of the aquifer. Head losses will also arise as water flows through the well screen; these losses will increase through time unless the well screen is maintained in pristine condition over its life. Finally, there will generally be some additional head losses within the well itself.

A conceptual model of the aquifer that assumes that confined storage is the only source of water may be reasonable for a relatively brief test in which the effects of pumping have not propagated far beyond the pumping well. However, this conceptual model is not realistic when considering long-term conditions. In the long-term, the supplies for a production well do not come from confined storage. Long-term sources of supply include leakage from overlying aquifers across confining units, capture of water from surface water sources, and capture of recharge at the water table (Theis, 1940). None of these sources are considered in the Q_{20} calculation. There may also be barriers to flow, including the truncation of an aquifer against less permeable material. In effect, the Q_{20} method requires the extrapolation of the observations made during pumping into a period during which the assumptions of the underlying conceptual model will almost certainly be violated to a significant extent.

It is impossible to generalize whether the Q_{20} method will always overestimate or underestimate the safe yield of a well. The direction of the errors arising from application of the Q_{20} method will depend on the characteristics of each particular site. However, it is possible to state that the Q_{20} method will almost always yield estimates that are wrong. The errors reflect the fact that the assumptions that underlie the Q_{20} method are inappropriate over the long-term. As will be shown in the case study, for typical settings in the glaciated Great Plains, the errors arising from application of the Q_{20} method may be -significant.

5.4 The Modified Moell method

Maathuis and van der Kamp (2006) have proposed an alternative to the Q_{20} method, the Modified Moell method. According to the Modified Moell method, the safe yield for 20 years of pumping is given by:

$$Q_{20} = 0.7 \times \frac{Q\,H_A}{s_{100\,min} + \left(s_{20\,yrs} - s_{100\,min}\right)_{theo}} \qquad (5\text{-}6)$$

Here:

Q_{20} denotes the safe yield for 20 years of pumping;
Q denotes the actual discharge rate during the pumping test;
H_A denotes the available drawdown;
$s_{100\,min}$ denotes the drawdown observed after 100 minutes of pumping;
$s_{100\,min\text{-}theo}$ denotes the calculated theoretical drawdown after 100 minutes of pumping; and
$s_{20\,yrs\text{-}theo}$ denotes the calculated theoretical drawdown after 20 years of pumping.

The Modified Moell method is founded firmly in both observations made during the pumping test and on a defensible strategy for extrapolation beyond the duration of the test. Equation (5-6) is straightforward to interpret. The Modified Moell method can be expressed in general form as:

$$Q_{long-term} = FS \times SC \times H_A \qquad (5\text{-}7)$$

The factor of safety, *FS*, corresponds to the leading coefficient of 0.7. This is the same value as was adopted for the Farvolden Q_{20} calculation. The term *SC* corresponds to the long-term *specific capacity, SC,* of the well:

$$SC = \frac{Q}{s_{100\,min} + \left(s_{20\,yrs} - s_{100\,min}\right)_{theo}} \qquad (5\text{-}8)$$

The safe yield of the well is calculated by multiplying the allowable drawdown H_A by the specific capacity.

It has been observed that the additional sources of drawdown in the pumping well that have been described previously are established quickly and remain constant through time (see for example, Walton, 1970; Herbert and Barker, 1995). Therefore, an appropriate proxy for the additional wells losses is the observed drawdown after 100 minutes of pumping, $s_{100\,min}$. The quantity $(s_{20\,years} - s_{100\,min})_{theo}$ represents the drawdown that would occur in the formation beyond 100 minutes of pumping. In contrast to the Farvolden Q_{20} method, no specific conceptual model is assumed. As indicated in the Alberta Environment *Guide to Groundwater Authorization*, the theoretical calculation is based on the appropriate model for the aquifer. In the case study, a model of a buried channel aquifer is used for the calculation.

Drawdowns in the pumping well observed during a pumping test conducted in southern Cambodia are used to illustrate the interpretation of the specific capacity. As shown in Figure 5-4, most of the drawdown occurs within the first minute of observations. The drawdown after 2 minutes is about 2.35 m, compared with a drawdown at the end of the test of 2.75 m. These data demonstrate that the head losses in the formation represent a relatively small fraction of the observed drawdown.

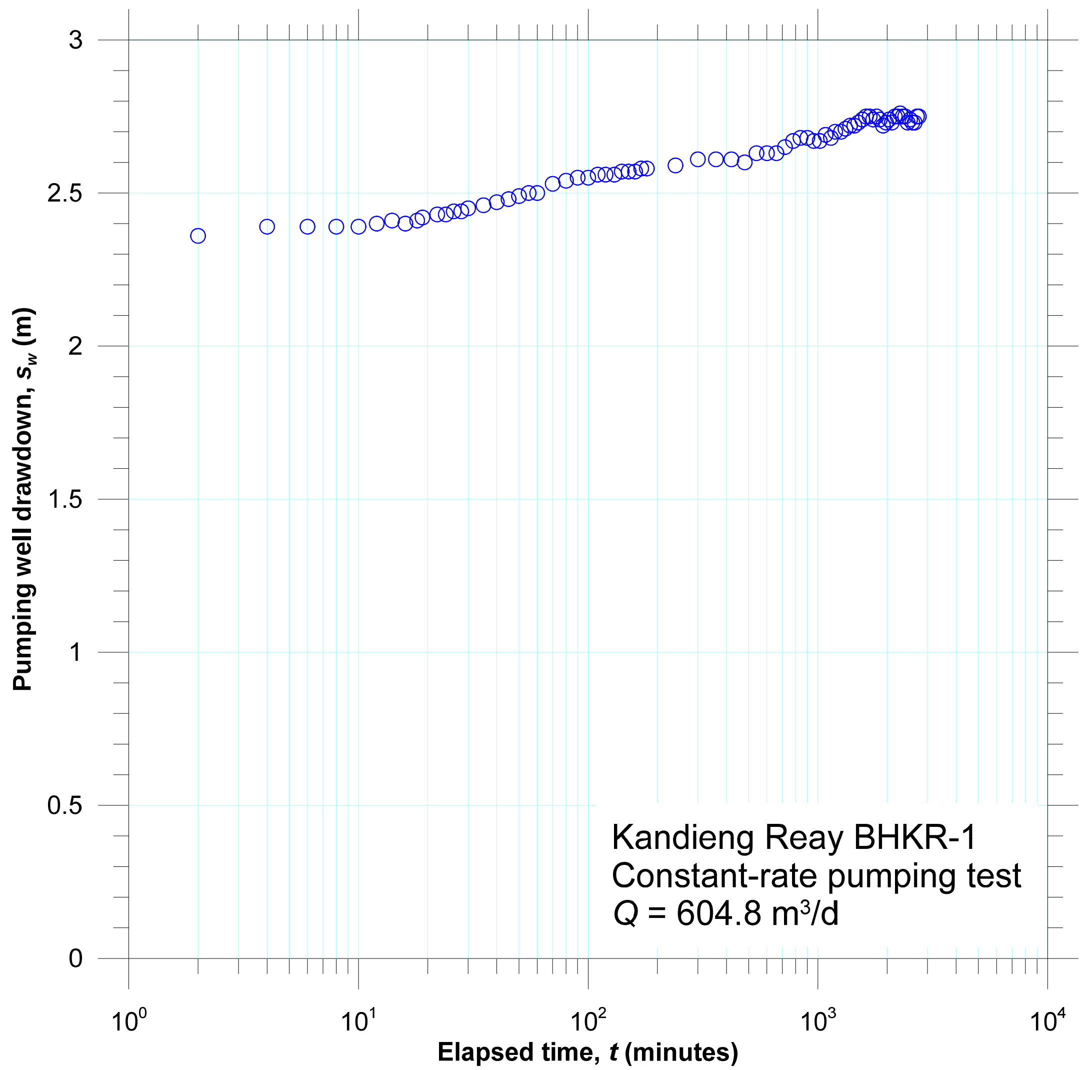

Figure 5-4. Pumping well drawdowns observed during a 48-hour pumping test.

The observed drawdown after 100 minutes, $s_{100\ min}$, is 2.55 m. To estimate the drawdown after 20 years, a simple conceptual model is adopted that is consistent with the limited available data:

$$s_w(t) = \frac{Q}{4\pi T} W\left(\frac{r_w^2 S}{4Tt}\right) + CQ^2 + \frac{Q}{4\pi T} 2S_w \qquad (5\text{-}9)$$

The first quantity represents head losses in the formation, with the aquifer modeled as an ideal confined aquifer with the Theis (1935) solution. The second term represents the additional nonlinear well losses. The nonlinear well coefficient, *C*, is estimated from the results of a step test conducted on the pumping well before the start of the constant-rate test. The third term represents the additional losses across a zone of altered material around the well. The well was drilled with a mud rotary rig, and it is reasonable to assume that the invasion of drilling mud into the formation has resulted in a localized reduction of the hydraulic conductivity of the aquifer. The skin loss is characterized by the dimensionless skin loss coefficient, S_w (Ramey, 1982). The value of S_w is also estimated from the results of the step test.

Referring to Figure 5-5, the theoretical drawdown after 20 years of pumping is 2.54 m; and the theoretical drawdown after 20 years of pumping is 3.27 m.

Therefore, the specific capacity is given by:

$$SC = \frac{Q}{s_{100\ min} + \left(s_{20\ yrs} - s_{100\ min}\right)_{theo}} = \frac{(604.8\ m^3/d)}{2.54\ m + (3.27\ m - 2.55\ m)} = 185\ m^3/d/m$$

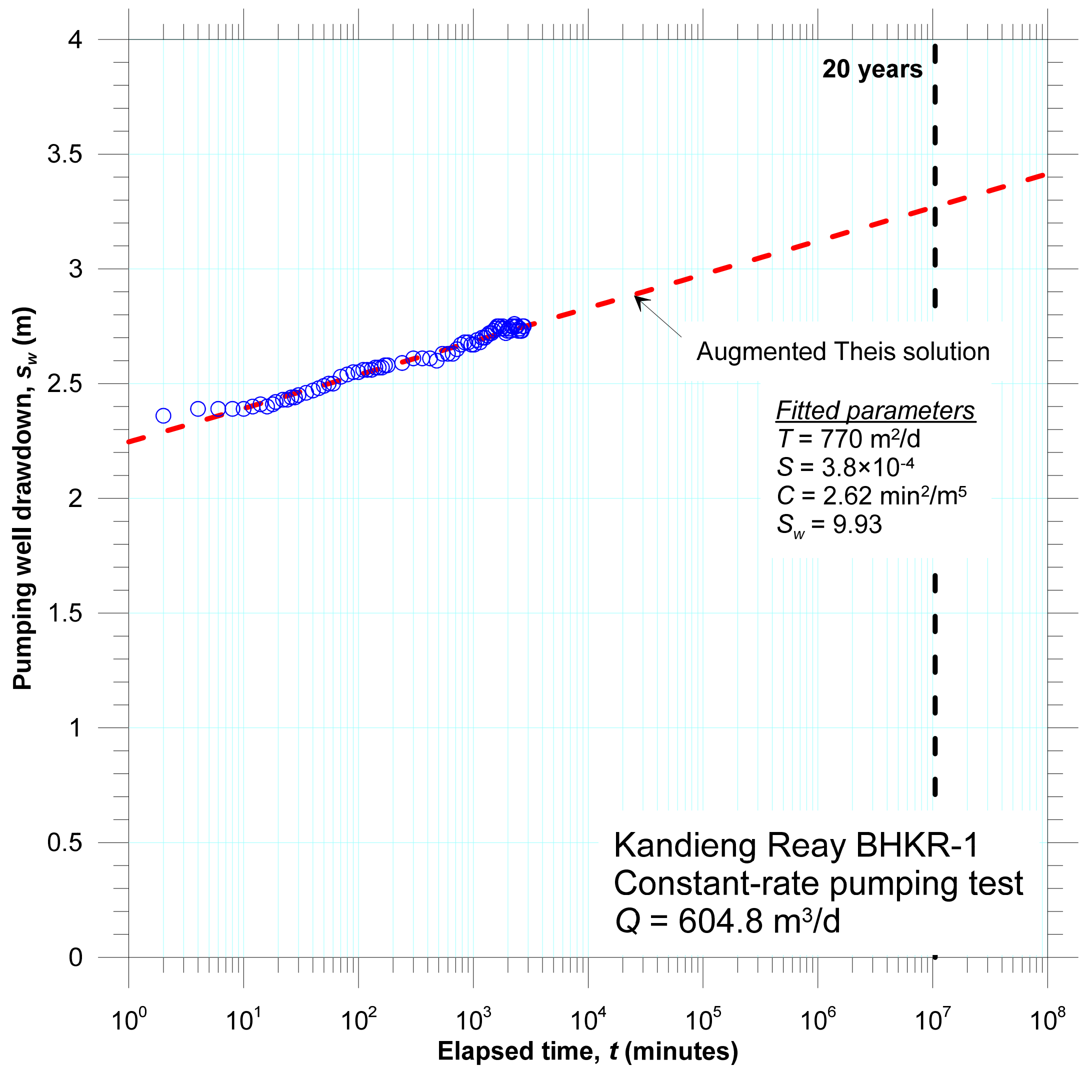

Figure 5-5. Match of theoretical solution to the observations and extrapolation to 20 years of pumping.

The last component of the Modified Moell calculation is the interpretation of the available drawdown, H_A. According to the *Guide to Groundwater Authorization* (Alberta Environment, 2011), the available drawdowns for confined and unconfined aquifers are defined as:

- Confined aquifer: the difference between the non-pumping water level and the elevation of the top of the aquifer; and

- Unconfined aquifer: two-thirds of the initial saturated thickness of the aquifer.

The rationale for the definition of the allowable drawdown for a confined aquifer is clear. The definition of the allowable drawdown in an unconfined aquifer is somewhat consistent with guidance presented in *Groundwater and Wells* (Driscoll, 1986; p. 433-434). It is indicated in *Groundwater and Wells* that for a homogeneous unconfined aquifer, screening of the bottom one-third to one-half of an aquifer less than 150 ft thick (45 m) is optimal. It is further indicated that an unconfined aquifer is usually pumped so that, at maximum capacity, the pumping water level is maintained slightly above the top of the screen. As shown in Figure 5-6, the combination of these two pieces of guidance leads to the definition of the allowable drawdown as ranging between two-thirds and one-half of the initial saturated thickness of the aquifer.

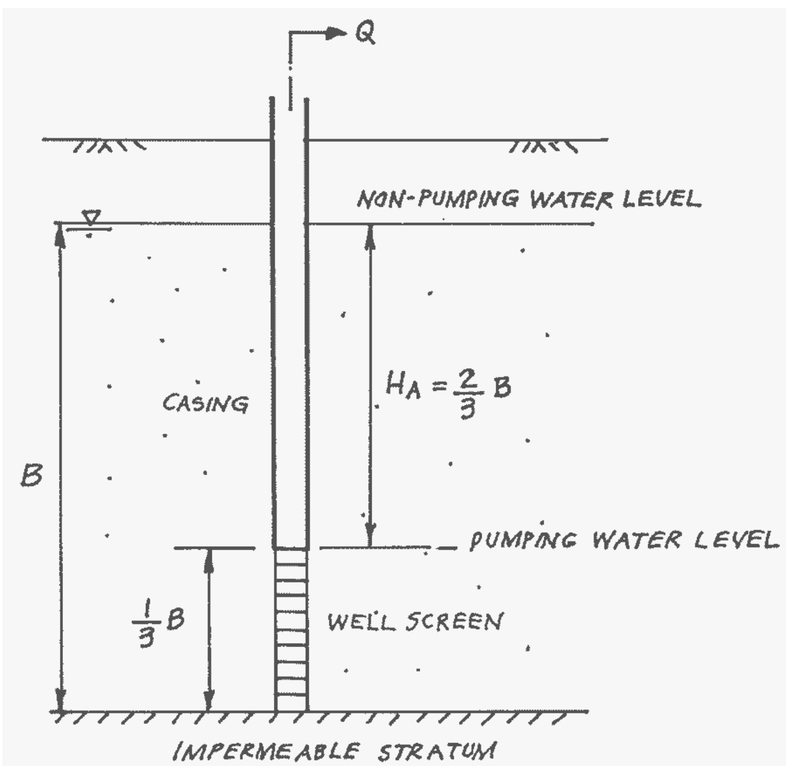

Figure 5-6. Definition of the allowable drawdown for an unconfined aquifer.

The Cambodian production well BHKR-1 is located in a confined fine to medium sand aquifer. The following key information is drawn for the report on the testing of the well (Cambodia Construction & Engineering Co., Ltd., 2005):

- The static groundwater level is 3.85 m below the measuring point;
- The top of the aquifer is located 19.9 m below the measuring point; and
- The top of the well screen is located 23 m below the measuring point.

The maximum allowable drawdown is interpreted here as the difference between the static groundwater level and the top of the aquifer, 17.9 m – 3.85 m = **14.07 m**. The drawdown to the top of the well screen is about 19.2 m. The minimum pumping level is therefore safely above the top of the well screen. Substituting into the Modified Moell formula yields the following estimate for the safe yield of the well:

$$
\begin{aligned}
Q_{long-term} &= FS \times SC \times H_A \\
&= (0.7) \times (185\ \mathrm{m^3/d/m}) \times (14.07\ \mathrm{m}) \\
&= \underline{\mathbf{1820\ m^3/d}}
\end{aligned}
$$

6. APPROPRIATE REPORTING OF PUMPING TESTS

There are many reasons why a pumping test must be conducted, and the specific objectives for each test should determine the level of reporting. Furthermore, the interpretation of pumping tests necessarily involves professional judgment. Therefore, a single prescriptive template is not recommended for pumping test reports. Although reporting based on a template is discouraged, sufficient information should be provided such that an independent reviewer can reproduce the analysis. More generally, upon reaching the end of a pumping test report, the reader should have a clear sense that:

1. The data are reliable;
2. The interpretations are internally consistent; and
3. The interpretations are consistent with everything else that is known about the site.

6.1 Minimum Elements Of The Reporting Of A Pumping Test

A prescriptive template is not recommended for the reporting of a pumping test, but the documentation of a pumping test must include certain minimum elements. These minimum elements are assembled in the form of a set of questions.

1. Why was the test conducted?
2. Were the objectives of the test accomplished?
3. What is the regional setting (hydrology, geology, hydrostratigraphy)?
4. How was the pumping rate controlled and measured, and how were those measurements checked?
5. How were water level changes observed during the test?
6. Where did the pumped water go? Was the location of the discharge sufficiently distant that water was not recirculated?
7. What was the basis for setting the rate for the constant-rate pumping test?
8. Were the pressure transducers checked *in situ*?
9. Did the recorded pressure changes remain within the manufacturer's recommended ranges?
10. Were the measurements with the transducers consistent with the results of manual measurements with an electric water level tape?
11. How were the observations processed to estimate the changes in water levels due only to pumping?
12. Were the results of the step test interpreted to enhance the understanding of the characteristics of the pumping well?
13. Are the interpretations of the step test consistent with the interpretations of the constant-rate pumping test?
14. Does the conceptual model adopted for the final analysis of the pumping test yield a consistent set of parameters?
15. Is the inferred conceptual model consistent with everything else that is known about the site and its regional setting?
16. Can the interpretation of the pumping test be used to support predictions of the long-term yield of the pumping well?
17. Can the interpretation of the pumping test support predictions of the potential effects of pumping on other water takers and on ecological features?

18. Are the results obtained from drawdown and recovery data consistent?

6.2 Appropriate Reporting Of Pumping Test Interpretations

Hydrogeology is an interpretive discipline. When conveying the results of any hydrogeologic analysis, it is important that the fundamental aspects of the discipline are conveyed to the reader. The structure of an aquifer and the distribution of material properties are essentially unknowable. A pumping test yields insights regarding the bulk-average properties of the aquifer. Because of the diffusive characteristics of groundwater flow, a pumping test does not provide much information on the existence of preferential pathways. Therefore, it is important to report the results of pumping tests appropriately.

When reviewing pumping test reports, it is common to see examples of reporting that suggest the analyst either does not understand the difference between precision and accuracy, or has an erroneous sense of exactitude. For example, the transmissivity calculated from a pumping test analysis might be reported as 2,105 m^2/day. This reporting is too exact for a groundwater application. If the analysis is well-constrained, the transmissivity may be reported is 2,100 m^2/d; however, it is generally more appropriate to report the estimate as "about 2,100 m^2/d".

Finally, it should be clear in the reporting that the parameter values are estimates and <u>not</u> facts. It is indicated frequently in hydrogeology that a particular quantity has been "determined". Hydrogeologists do not determine anything. Rather, they estimate and infer. Hydrogeologic analyses are always provisional and the results are contingent upon the current understanding of the conceptual model that has been invoked to interpret the data.

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Appendix 1

Checklists for the Execution of Pumping Tests

Overview

Several manuals describing field hydrogeology practice are available (Sanders, 1998; Brassington, 2007, Weight, 2008, Moore, 2012). Three compact checklists have been developed to assist in the execution of pumping tests:

- What to do before you travel to the site;
- What to do at the site; and
- What to do at the end of the test.

Checklist #1

What to do before you travel to the site		Check
1	Design your test.	
2	Plan the test to induce the maximum stress possible on the aquifer, but avoid drawing the water level below the top of a confined aquifer, and avoid pumping sand.	
3	Identify the techniques that will provide accurate measurements of elapsed time, distances between wells, drawdown, and pumping rate. • Record time with synchronized digital watches and a stopwatch. • Include plans to measure water levels manually every time a transducer is downloaded. • Include provisions to control and measure the pumping rate.	
4	Select transducers that cover the anticipated range of water level changes.	
5	Check that water level tapes, transducers and dataloggers are working properly.	
6	Know whether the transducers are vented or non-vented. If the transducers are non-vented, decide where to install one of the transducers as a barometer.	
7	Check that the cumulative flowmeter has been calibrated and is working properly.	
8	Ensure discharge is managed and disposed of in accordance with regulations (especially for contaminated water)	
9	Ensure discharge is sufficiently isolated or remote to avoid recharging aquifer	

Checklist #2

What to do at the site		Check
1	Locate the generator away from the pumping well to reduce the background noise.	
2	Calibrate the transducers in the field as they are being installed in each well.	
3	Set the references on the dataloggers so that the output can be compared directly with the depth to water measured with a water level tape	
4	For systems that consist of separate dataloggers and transducers, avoid immersing dataloggers, as they may not be watertight. Avoid electric shocks. When downloading data, touch the cable and your hands to the screws on the top of the datalogger before connecting the cable to the transducer.	
5	Perform pre-test monitoring to determine the magnitudes of water level variations due to external influences.	
6	Check that the pump intake is sufficiently deep to allow adequate drawdown.	
7	Place a check valve on the discharge pipe above the pump.	
8	• If the pumping well does not produce sand, measure the pumping rate with a totalizing flowmeter and control the rate with a valve and an in-line flowmeter. • If the pumping well does produce sand, consider alternative approaches for measuring the discharge, e.g., an orifice tube, weir box, bucket-and-stopwatch	
9	Conduct a step test to identify the sustainable rate for a constant-rate pumping test. Choose pumping rates that can be monitored easily.	
10	After conducting the step test, allow sufficient time for water levels in the aquifer to recover completely	

Checklist #3

What to do at the end of the test		Check
1	Backup downloaded data. Backup backed-up downloaded data.	
2	Secure the transducers.	
3	Continue monitoring recovery for as long as possible.	
4	• Plot the pumping rate versus time. • Plot water levels versus time for the pumping well and the observation wells.	

Appendix 2

Diagnostic Responses to Pumping

Overview

When the interpretation of a pumping test is presented as a step-by-step procedure for matching a theoretical model to drawdown observations, it is easy to lose fact of the site that this is not the analyst's only activity. *Quantification* of aquifer properties is important, but the key aspect of analysis is the inference of the conceptual model that captures the essential elements of a site. This is the process of *diagnosis*. Diagnosis is to some extent pattern recognition. To assist pumping test interpreters, log-log and semi-log plots of both the drawdown and the drawdown derivative have been assembled for several common conceptual models, with indications of key diagnostic features of the individual plots.

The following conceptual models are considered.

1. Theis aquifer
2. Theis aquifer, wellbore storage
3. Theis aquifer, partial penetration
4. Theis aquifer, linear recharge boundary
5. Theis aquifer, linear no-flow boundary
6. Theis aquifer, two linear no-flow boundaries

The following notes apply to the figures:

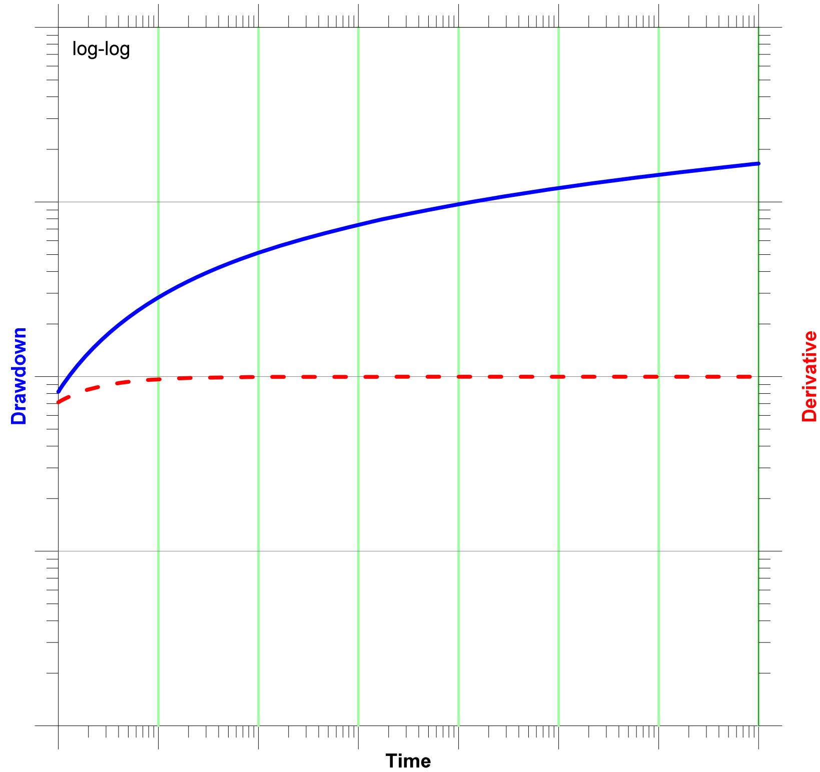
- "Plateau" refers to the beginning of the portion of the derivative plot during which the derivative is approximately constant; and.
- "CJSL" refers to the portion of the response over which the Cooper-Jacob straight-line analysis should be applied.

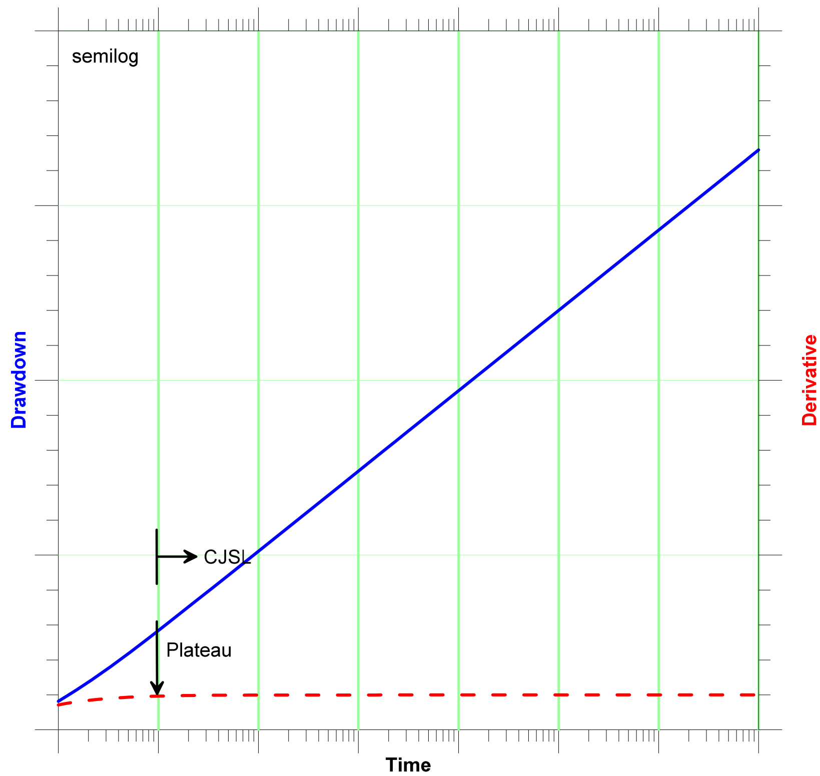
Renard and others (2009) present a good introduction to derivative analysis from a hydrogeology perspective, with a more extensive compilation of derivative plots.

Reference

Renard, P., D. Glenz, and M. Mejias, 2009: Understanding diagnostic plots for well-test interpretation, *Hydrogeology Journal*, vol. 17, pp. 589-600.

1. THEIS (1935) MODEL



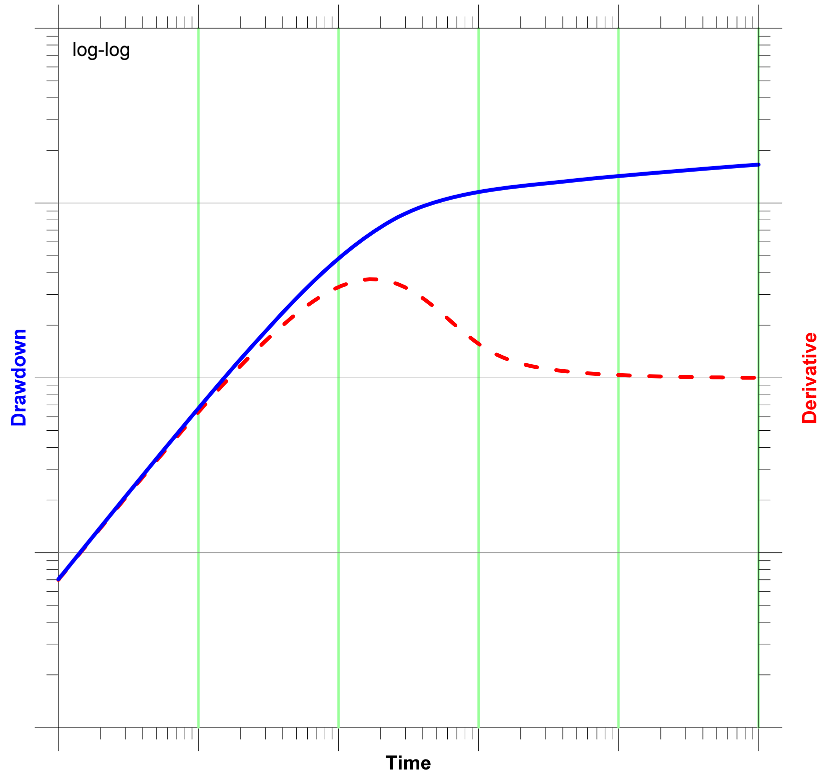


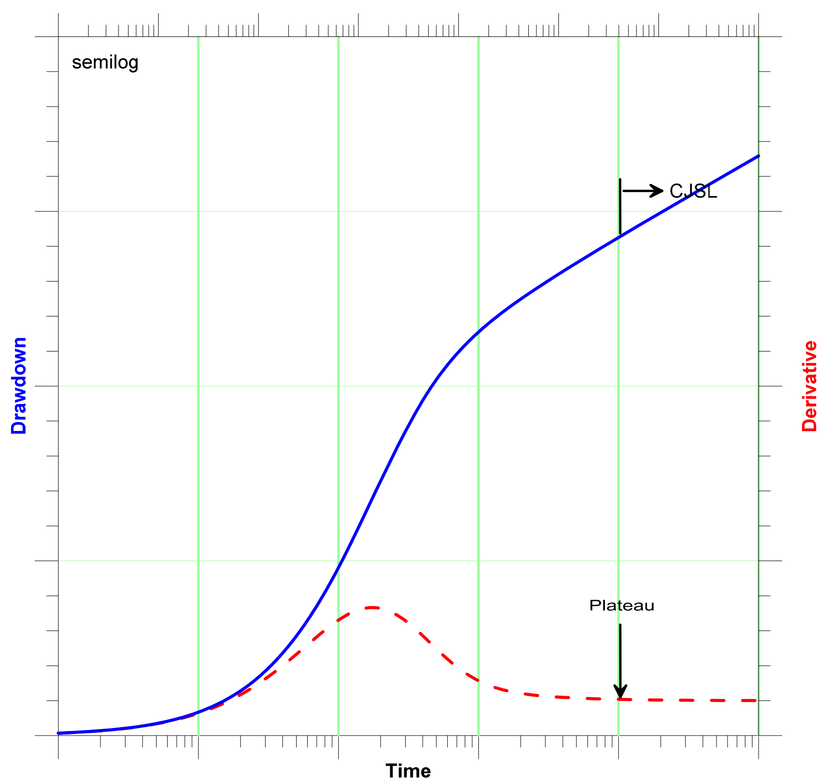
Diagnosis of response following the Theis model

For intermediate time:

- Drawdowns on the semi-log plot approximate a straight line; and
- Drawdown derivative reaches a constant value, referred to here as a plateau.

2. THEIS (1935) MODEL, WELLBORE STORAGE





Diagnosis of response following the Theis model with wellbore storage

For early time:

- Drawdowns on log-log plot approximate a straight line with a unit slope; and
- Drawdown and drawdown derivative are the same.

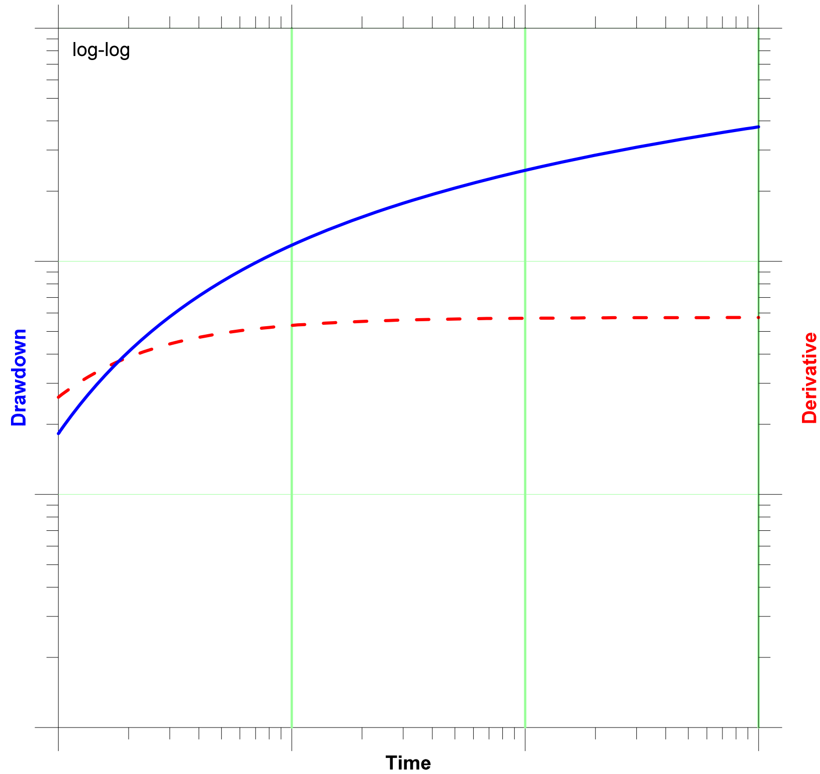
For intermediate time:

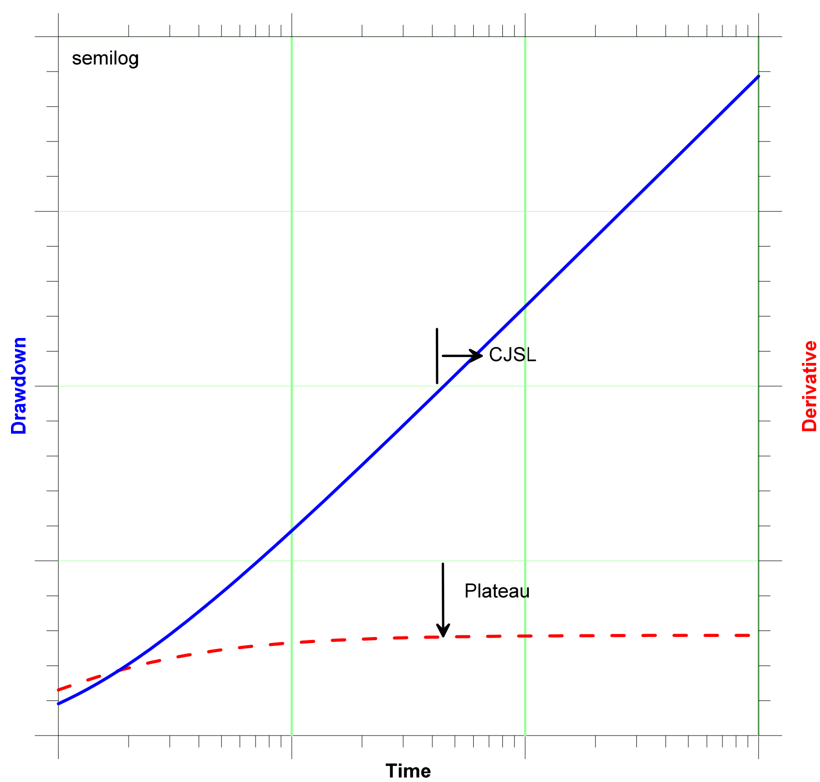
- Drawdown has a "hump".

For later time:

- Drawdowns on the semi-log plot approximate a straight line; and
- Drawdown derivative reaches a constant value.

3. THEIS AQUIFER, PARTIAL PENETRATION



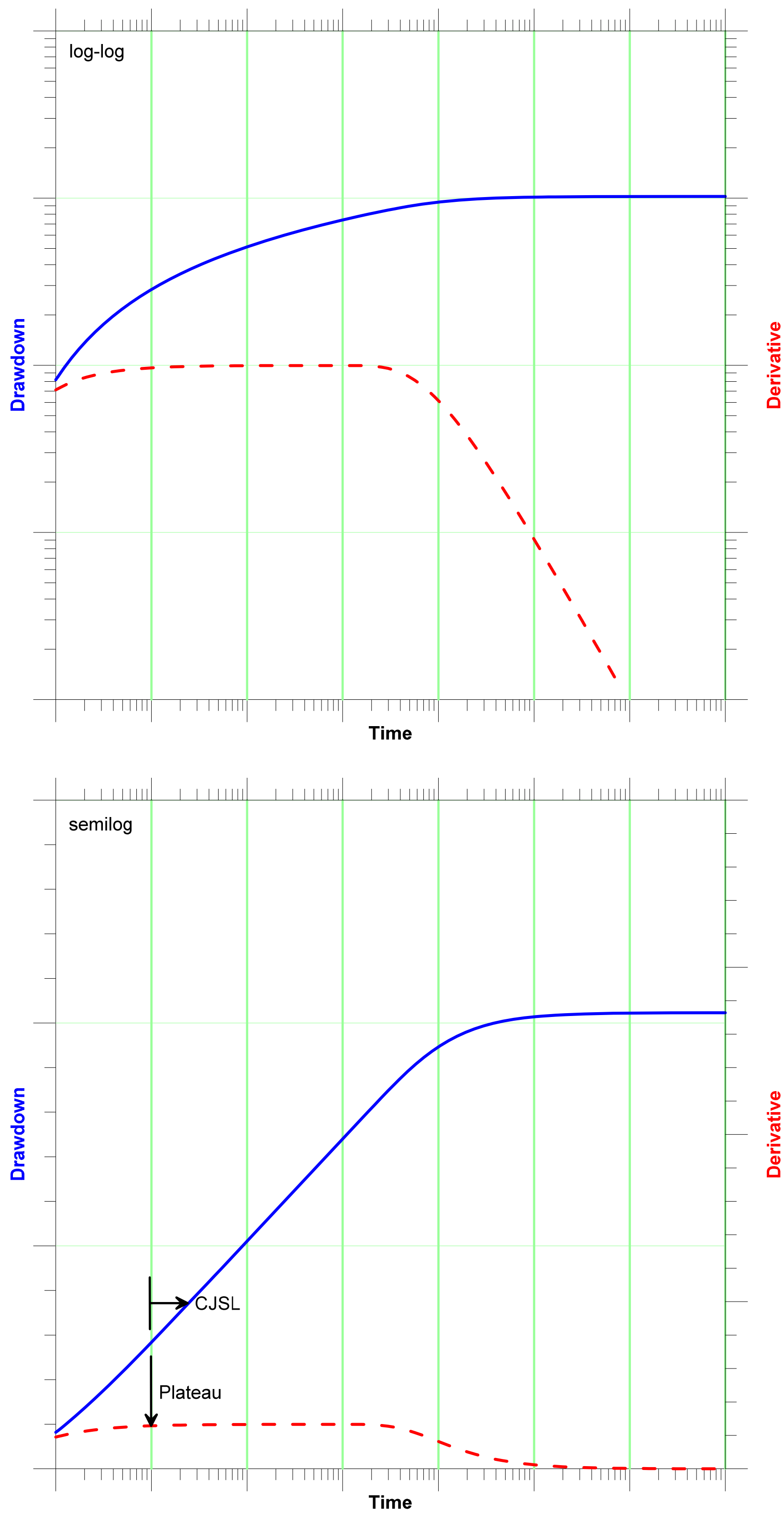


Diagnosis of response following the Theis model for a partially penetrating well

For intermediate time:

- Drawdowns on the semi-log plot approximate a straight line; and
- Drawdown derivative reaches a constant value.

4. THEIS AQUIFER, LINEAR RECHARGE BOUNDARY



Diagnosis of response following the Theis model with a linear recharge boundary

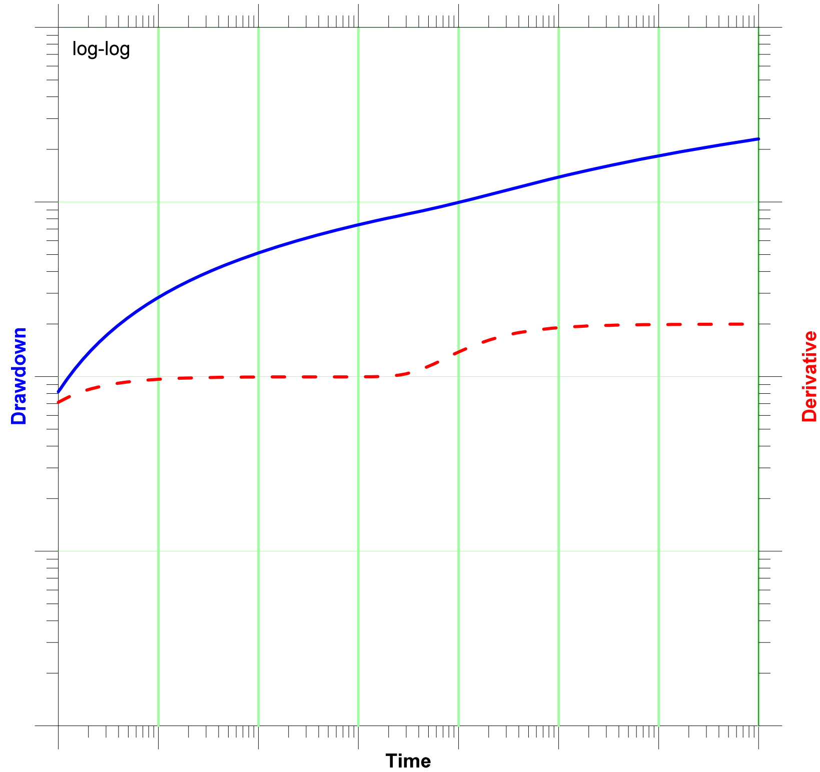
For intermediate time:

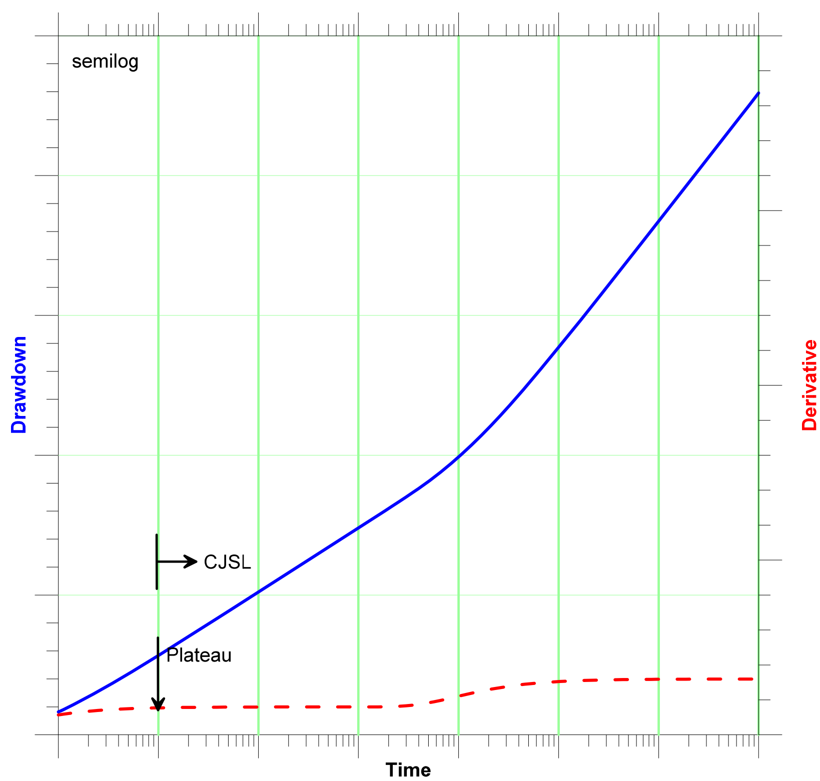
- Drawdowns on the semi-log plot approximate a straight line; and
- Drawdown derivative temporarily stabilizes at a constant (non-zero) value.

For later time:

- Drawdowns reach a constant value;
- Drawdown derivative decreases to 0.0; and
- Drawdown derivative decreases linearly on the log-log plot.

5. THEIS AQUIFER, LINEAR NO-FLOW BOUNDARY





Diagnosis of response following the Theis model with a linear no-flow boundary

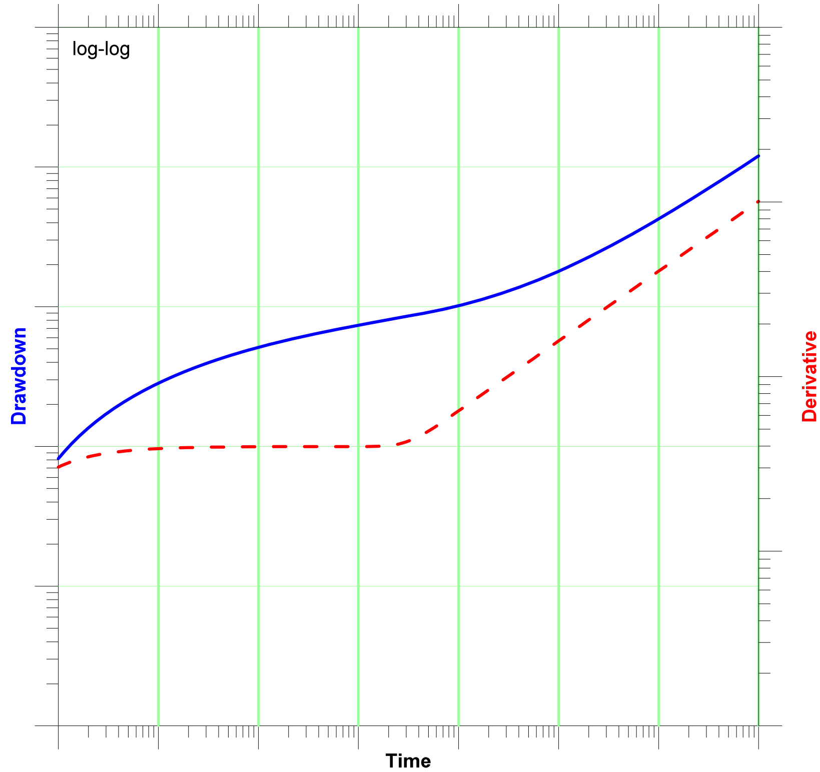
For intermediate time:

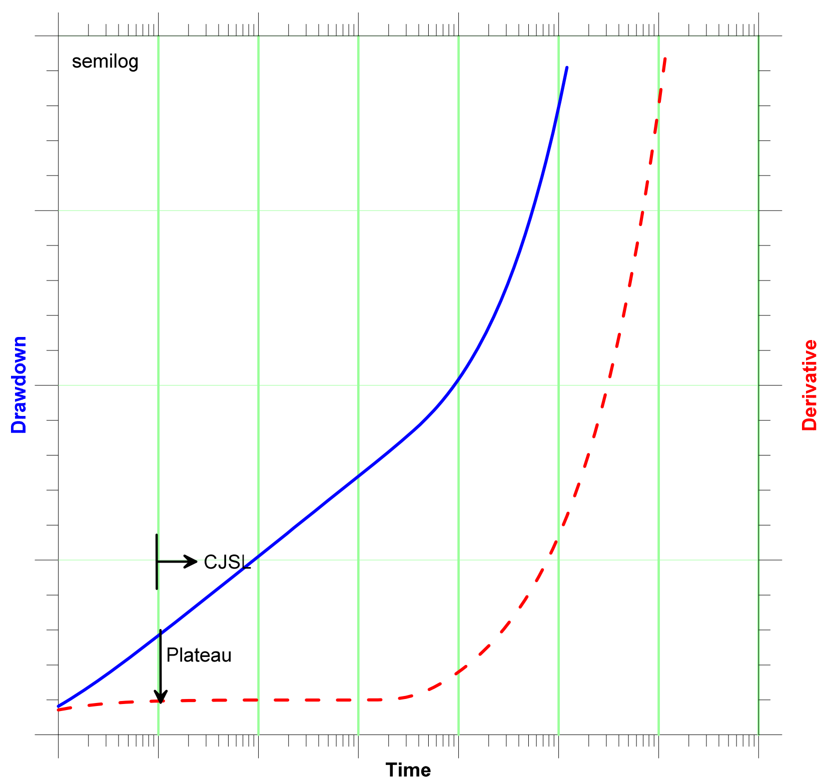
- Drawdowns on the semi-log plot approximate a straight line; and
- Drawdown derivative temporarily stabilizes at a constant (non-zero) value.

For later time:

- Drawdowns on the semi-log plot approximate a second straight line, with a slope that is double of that for intermediate time; and
- Drawdown derivative stabilizes at a value that is double the value from intermediate time.

6. THEIS AQUIFER, TWO LINEAR NO-FLOW BOUNDARIES





Diagnosis of response following the Theis model with a linear no-flow boundary

For intermediate time:

- Drawdowns on the semi-log plot approximate a straight line; and
- Drawdown derivative temporarily stabilizes at a constant (non-zero) value.

For later time:

- Drawdowns on the log-log plot approximate a straight line, with a half-slope (one log cycle of drawdown per two log cycles of time); and
- Drawdown derivative on the log-log plot approximates a straight line, with a half-slope.

Appendix 3

Fundamentals of Pressure Transient Analysis for Hydrogeologists

Overview

When hydrogeologists think about pumping tests, they generally do not refer to the petroleum engineering literature. This is unfortunate, as the petroleum engineering literature contains a wealth of detail on advanced techniques of analysis, and guidance for interpretation. It represents an important limitation in an Alberta context, as many pumping tests are conducted to support oil and gas development. The apparent unfamiliarity of hydrogeologists with key references in the petroleum engineering literature may reflect the fact that hydrogeologists and petroleum engineers speak related but distinct languages.

The materials that follow are devoted to a presentation of the fundamentals of the interpretation of well tests from the perspective of petroleum engineering. They are intended to provide a "bridge" between the disciplines of hydrogeology and petroleum engineering. As far as the author is aware, no attempt has been made previously to establish this link. The development of these materials has been motivated by a desire to make accessible to hydrogeologists the important developments of well testing that have been made, and continue to be made, by petroleum engineers. In addition to reviewing the terminology, two foundational analyses are developed from first principles. A relatively large number of equations is presented. This is deliberate; the intention is to show in detail the relations between the theory of well tests as conceived by petroleum engineers and hydrogeologists and thereby eliminate any potential mysteries that may arise from differences in terminology and notation.

The materials are divided into seven main sections:

1. Introduction;
2. Terminology;
3. Governing equations for pressure transient analysis and pumping test interpretation;
4. Analytical solution for the "Line-Sink" problem – The Theis solution;
5. Analysis of the recovery following pumping from a "Line-Sink" – The Horner/Cooper-Jacob analysis;
6. Summary; and
7. Selected references for further reading.

1. INTRODUCTION

The CEMA Complementary Information for Water Well Testing has been prepared by hydrogeologists and the focus is directed to groundwater practice. There are at least three reasons why it is important to include some discussion of the fundamentals of petroleum engineering practice with respect to the interpretation of well tests.

First, there has historically been significant cross-over between the hydrogeologic and petroleum engineering practice with respect to well test analysis. It is important for hydrogeologists to be able to read and appreciate the petroleum engineering literature because it remains a rich source of new solutions and interpretation techniques. This must start with recognition that petroleum engineers have developed many of the conceptual models and solutions that hydrogeologists use in their analyses. This recognition should direct hydrogeologists to the original sources in the petroleum engineering literature. As just one example, French petroleum engineers introduced derivative analysis (Bourdet and others, 1983; Bourdet and others, 1989).

Second, there is also much that hydrogeologists can glean from several excellent review articles, monographs and textbooks that summarize established practice and advanced approaches in petroleum engineering practice (for example, Earlougher, 1977; Horne, 1995; and Bourdet, 2002).

Finally, in the context of professional practice in Alberta, there is a strong possibility that hydrogeologic tests are conducted to support the development of oil and gas resources, and are conducted under the direction of petroleum engineers. Therefore, it is important that hydrogeologists and petroleum engineers have a common understanding for the design and interpretation of these tests.

2. TERMINOLOGY

2.1 Pressure Transient Analysis And Hydrogeologic Testing

Several terms have been used in the petroleum engineering literature to describe well testing and interpretation. These terms include:

- Pressure analysis (e.g., SPE Reprint Series No. 9);
- Well testing and analysis (Earlougher, 1977; Thorne, 1995; Bourdet, 2002); and
- Pressure transient testing and analysis (e.g., SPE Reprint Series No. 14, No. 57, and Kappa, 2011).

Varied terms have also been used in hydrogeologic practice, including pumping tests and aquifer tests. However, it is important to note that many tests are not conducted by pumping a well, and many tests are conducted in materials that do not yield economic quantities of potable water and therefore cannot be considered as aquifers. For maximum generality, the term *pressure transient analysis* will be used in the remainder of this section of the guidelines. Since attention is restricted to tests involving the pumping of groundwater, the hydrogeologic equivalent will be *pumping test*.

2.2 Pressure Change And Drawdown

In pressure transient analysis, the key quantities that are recorded are fluid discharge and change in fluid pressure. The pressure change, Δp, is defined as:

$$\Delta p(r,z,t) = p_i(r,z,t) - p(r,z,t) \quad (1)$$

where $p_i(r,z,t)$ is the pressure that would be observed at a distance r from a production well and elevation z if no pumping had occurred, and $p(r,z,t)$ is the pressure that is actually observed. It is assumed frequently that the pressure at the start of the test is constant everywhere and that there are no changes in the background pressure during the test, such that $p_i(r,z,t)$ is replaced by a constant p_i (see Earlougher, 1977; Equation 2.2). However, this assumption is unnecessarily restrictive.

The fluid discharge is also recorded in pumping tests, and is referred to as the *pumping rate*; however, the effects of pumping are generally reported in terms of the change in hydraulic head, referred to as *drawdown*.

The hydraulic head is defined as:

$$h(r,z,t) = \frac{p(r,z,t)}{\rho_w g} + z \quad (2)$$

where ρ_w is the density of water and g is the acceleration due to gravity. The hydraulic head is meaningful only when the density of water is constant.

The drawdown is defined as:

$$\begin{aligned} s(r,z,t) &= h_i(r,z,t) - h(r,z,t) \\ &= \left[\frac{p_i(r,z,t)}{\rho_w g} + z\right] - \left[\frac{p(r,z,t)}{\rho_w g} + z\right] \\ &= \frac{\Delta p(r,z,t)}{\rho_w g} \end{aligned} \quad (3)$$

Working in terms of pressure is more fundamental, as the assumption of constant fluid density is not invoked. However, the variations in density must be relatively small for the data from a test to be tractable for interpretation with an analytical solution. If this is the case, the drawdown and pressure change differ only by the unit weight of water, $\rho_w g$. For tests in which the density varies significantly in space and/or in time, it is appropriate to use a numerical simulator that can handle these variations.

3. GOVERNING EQUATIONS FOR PRESSURE TRANSIENT ANALYSIS AND PUMPING TEST INTERPRETATION

3.1 Statement Of Mass Conservation

The general statement of mass conservation is:

$$\frac{\partial}{\partial t}(\phi\rho) = -\frac{1}{r}\frac{\partial}{\partial r}\left[r\rho q_r\right] - \frac{\partial}{\partial z}\left[\rho q_z\right] \qquad (4)$$

where ρ is the fluid density, ϕ is the porosity of the porous medium, and q_r and q_z are the radial and vertical Darcy fluxes, respectively. For simplicity, it is assumed that the material properties do not vary with respect to the direction from the pumping well. An analytical approach is available for the case of a uniform but anisotropic porous medium (Papadopulos, 1965).

3.2 Darcy Fluxes

The Darcy fluxes are given by Darcy's Law, which are expressed in their most general form as (Bear, 1972: Equation 4.7.17):

$$q_r = -\frac{k_r}{\mu}\left(\frac{\partial p}{\partial r}\right) \qquad (5)$$

$$q_z = -\frac{k_z}{\mu}\left(\frac{\partial p}{\partial z} + \rho g\right) \qquad (6)$$

where μ is the dynamic viscosity of the fluid, and k_r and k_z are the radial and vertical permeabilities, respectively.

3.3 The Governing Equation In The Petroleum Engineering Literature

Substituting for the Darcy fluxes from (5) and (6) into (4) yields:

$$\frac{\partial}{\partial t}(\phi\rho) = -\frac{1}{r}\frac{\partial}{\partial r}\left[r\rho\left[-\frac{k_r}{\mu_w}\left(\frac{\partial p}{\partial r}\right)\right]\right] - \frac{\partial}{\partial z}\left[\rho\left[-\frac{k_z}{\mu_w}\left(\frac{\partial p}{\partial z} + \rho g\right)\right]\right] \qquad (7)$$

The left-hand side of Equation (7), referred to as the *mass accumulation term*, is expanded as:

$$\frac{\partial}{\partial t}(\phi\rho) = \phi\frac{\partial\rho}{\partial p}\frac{\partial p}{\partial t} + \rho\frac{\partial\phi}{\partial p}\frac{\partial p}{\partial t} \qquad (8)$$

The following constitutive relations are assumed to govern the compressibility terms:

$$\rho = \rho_0 \; EXP\left\{c_f \left(p - p_0\right)\right\} \quad \rightarrow \quad \frac{\partial \rho}{\partial p} = c_f \rho \tag{9}$$

$$\phi = \phi_0 \; EXP\left\{c_r \left(p - p_0\right)\right\} \quad \rightarrow \quad \frac{\partial \phi}{\partial p} = c_r \phi \tag{10}$$

where ρ_o and ϕ_o are the density and porosity at a reference pressure p_0, and c_f and c_r are the compressibility of the fluid and the porous medium skeleton, respectively. Substituting (9) and (10) into (8) yields:

$$\frac{\partial}{\partial t}\left(\phi\rho\right) = \phi c_f \rho \frac{\partial p}{\partial t} + \rho c_r \phi \frac{\partial p}{\partial t} \tag{11}$$

Collecting terms:

$$\frac{\partial}{\partial t}\left(\phi\rho\right) = \phi\rho\left(c_f + c_r\right)\frac{\partial p}{\partial t} \tag{12}$$

The quantity (c_f+c_r) is referred to as the *total system compressibility*, c_t. Substituting for c_t, the mass accumulation term is written as:

$$\frac{\partial}{\partial t}\left(\phi\rho_w\right) = \phi\rho c_t \frac{\partial p}{\partial t} \tag{13}$$

Substituting for the left-hand side in (7), and assuming constant fluid properties yields:

$$\phi\rho c_t \frac{\partial p}{\partial t} = \frac{\rho}{\mu}\frac{1}{r}\frac{\partial}{\partial r}\left[r\left[k_r\left(\frac{\partial p}{\partial r}\right)\right]\right] + \frac{\rho}{\mu}\frac{\partial}{\partial z}\left[\left[k_z\left(\frac{\partial p}{\partial z} + \rho g\right)\right]\right] \tag{14}$$

In general, if analytical solutions are sought for the governing equation it is necessary to assume that permeabilities are uniform, at least within an individual hydrostratigraphic unit. If this assumption is made, the governing equation reduces to:

$$\phi\mu c_t \frac{\partial p}{\partial t} = k_r \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial p}{\partial r}\right] + k_z \frac{\partial^2 p}{\partial z^2} \tag{15}$$

If the vertical components of flow are negligible, Equation (10) reduces to:

$$\phi\mu c_t \frac{\partial p}{\partial t} = k_r \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial p}{\partial r}\right] \tag{16}$$

Dividing through by the permeability:

$$\frac{\phi\mu c_t}{k_r}\frac{\partial p}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial p}{\partial r}\right] \tag{17}$$

This is the form of the governing equation for transient fluid flow that represents the starting point for most analytical treatments of pumping tests in the petroleum engineering literature (e.g., Horne, 1995; Equation 2.2). It is assumed that the units are consistent.

Earlougher (1977) presents a version of the governing equation that incorporates the additional assumption that the vertical components of flow are negligible, and in terms of *oilfield units*. It is worthwhile deriving this form of the equation since it appears so widely in the petroleum engineering literature.

The oilfield units are tabulated below.

Parameter	
time, t	hours
radial distance, r	ft
pressure, p	psi
permeability, k	md (millidarcies)
dynamic viscosity, μ	cp (centipoise)
total system compressibility, c_t	psi^{-1}

Wyckoff and others (1933) introduced the oilfield unit of permeability, the darcy. The darcy is defined as the permeability of a porous medium that allows the flow of 1 cm^3/s of fluid of 1 cp viscosity through an area of 1 cm^2, under a pressure gradient of 1 atm/cm (Muskat, 1937; p. 76). It can be shown that a permeability of 1 md is equal to 9.86923×10^{-16} m^2.

Substituting for the units in both sides of Equation (12) yields:

$$\frac{\phi\ \mu(\text{cp})\left[\frac{10^{-3}\ \text{kg/m-s}}{\text{cp}}\right]c_t\left(\text{psi}^{-1}\right)\left[\frac{\text{psi}}{6.894757\times10^{3}\ \text{Pa}}\right]}{k_r(\text{md})\left[\frac{9.86923\times10^{-16}\,\text{m}^2}{\text{md}}\right]}\frac{\partial p(\text{psi})\left[\frac{6.894757\times10^{3}\ \text{Pa}}{\text{psi}}\right]}{\partial t(\text{hr})\left[\frac{3600\ \text{s}}{\text{hr}}\right]}$$

$$=\frac{1}{r(\text{ft})\left[\frac{0.3048\ \text{m}}{\text{ft}}\right]}\frac{\partial}{\partial r(\text{ft})\left[\frac{0.3048\ \text{m}}{\text{ft}}\right]}\left[r(\text{ft})\left[\frac{0.3048\ \text{m}}{\text{ft}}\right]\frac{\partial p(\text{psi})\left[\frac{6.894757\times10^{3}\ \text{Pa}}{\text{psi}}\right]}{\partial r(\text{ft})\left[\frac{0.3048\ \text{m}}{\text{ft}}\right]}\right]$$

Simplifying:

$$2.815\times10^{8}\,\frac{\phi\;\mu(\mathrm{cp})c_t\left(\mathrm{psi}^{-1}\right)}{k_r(\mathrm{md})}\frac{\partial p(\mathrm{psi})}{\partial t(\mathrm{hr})}=7.421\times10^{4}\,\frac{1}{r(\mathrm{ft})}\frac{\partial}{\partial r(\mathrm{ft})}\left[r(\mathrm{ft})\frac{\partial p(\mathrm{psi})}{\partial r(\mathrm{ft})}\right]$$

This yields finally:

$$\frac{1}{0.0002637}\frac{\phi\mu c_t}{k_r}\frac{\partial p}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial p}{\partial r}\right] \tag{18}$$

This is identical to Equation 2.1 in Earlougher (1977). It is useful to retain all of the significant figures in the leading coefficient as a reminder that unit conversions are involved.

3.3 The Governing Equation In The Hydrogeology Literature

For completeness, and to confirm the equivalence of the developments of pressure transient analysis and hydrogeologic interpretation of pumping tests, the governing equation for the transient flow of water through a porous medium is developed. The development follows the same steps as were used to derive Equation (18).

The expressions from the Darcy fluxes, Equations (5) and (6) are first expressed in terms of hydraulic head instead of pressure:

$$\begin{aligned} q_r &= -\frac{k_r}{\mu_w}\left(\frac{\partial}{\partial r}\left[\rho_w g\left[h-z\right]\right]\right) \\ &= -\rho_w g\,\frac{k_r}{\mu_w}\frac{\partial h}{\partial r} \end{aligned} \tag{19}$$

$$\begin{aligned} q_z &= -\frac{k_z}{\mu_w}\left(\frac{\partial}{\partial z}\left[\rho_w g\left[h-z\right]\right]+\rho_w g\right) \\ &= -\rho_w g\,\frac{k_z}{\mu_w}\frac{\partial h}{\partial z} \end{aligned} \tag{20}$$

The subscripts "*w*" are used to indicate that the development is now restricted to water.

Substituting for the Darcy fluxes into the Statement of Mass Conservation, Equation (4), yields:

$$\frac{\partial}{\partial t}(\phi\rho)=-\frac{1}{r}\frac{\partial}{\partial r}\left[r\rho_w\left[-\rho_w g\,\frac{k_r}{\mu_w}\frac{\partial h}{\partial r}\right]\right]-\frac{\partial}{\partial z}\left[\rho_w\left[-\rho_w g\,\frac{k_z}{\mu_w}\frac{\partial h}{\partial z}\right]\right] \tag{21}$$

Rather than working in terms of the fundamental quantity permeability, k, hydrogeologists quantify the relative ease with which water moves through a porous medium in terms of a lumped quantity called the hydraulic conductivity, K:

$$K = \frac{\rho_w g}{\mu_w} k \tag{22}$$

The hydraulic conductivities in the radial and vertical directions are:

$$K_r = \frac{\rho_w g}{\mu_w} k_r \qquad K_z = \frac{\rho_w g}{\mu_w} k_z \tag{23}$$

Substituting for the hydraulic conductivities in (21) yields:

$$\frac{\partial}{\partial t}(\phi \rho_w) = \frac{1}{r}\frac{\partial}{\partial r}\left[r \rho_w K_r \frac{\partial h}{\partial r}\right] + \frac{\partial}{\partial z}\left[\rho_w K_z \frac{\partial h}{\partial z}\right] \tag{24}$$

The left-hand side of (24), the mass accumulation term, is expanded in terms of the hydraulic head, using the product and chain rules of differentiation:

$$\frac{\partial}{\partial t}(\phi \rho_w) = \phi \frac{\partial}{\partial p}(\rho_w)\frac{\partial p}{\partial h}\frac{\partial h}{\partial t} + \rho_w \frac{\partial}{\partial p}(\phi)\frac{\partial p}{\partial h}\frac{\partial h}{\partial t} \tag{25}$$

Hydrogeologists assume the following constitutive relations that govern the relations between the change of head and the change of volume of the pore water and the porosity of the porous medium:

$$\rho_w = \rho_{w0}\, EXP\left\{ -\beta_w \left(p - p_0 \right) \right\} \quad \rightarrow \quad \frac{\partial \rho_w}{\partial p} = \beta_w \rho_w \tag{26}$$

$$\phi = \phi_0 + \alpha \left(p - p_0 \right) \quad \rightarrow \quad \frac{\partial \phi}{\partial p} = \alpha \tag{27}$$

Here β_w denotes the compressibility of water and α denotes the compressibility of the aquifer skeleton.

The development of the mass accumulation term in the hydrogeology formulation is slightly different from that adopted in petroleum engineering. The treatment of the change in fluid density with pressure is similar, but the assumed relation between the porosity and fluid pressure is different. However, the final expressions for the compressibility of the aquifer skeletons differ only by a constant factor, the porosity, ϕ.

According to the chain rule of differentiation:

$$\frac{\partial p}{\partial h}=\frac{\partial}{\partial h}\left[\rho_w g\left(h-z\right)\right]=\rho_w g \tag{28}$$

Substituting for (26), (27) and (28) into Equation (25) yields:

$$\frac{\partial}{\partial t}(\phi\rho_w)=\phi\left(\beta_w\rho_w\right)\left(\rho_w g\right)\frac{\partial h}{\partial t}+\rho_w\left(\alpha\right)\left(\rho_w g\right)\frac{\partial h}{\partial t} \tag{29}$$

Collecting terms:

$$\frac{\partial}{\partial t}(\phi\rho_w)=\rho_w\rho_w g\left[\phi\beta_w+\alpha\right]\frac{\partial h}{\partial t} \tag{30}$$

Hydrogeologists define the specific storage, S_s as:

$$S_s=\rho_w g\left[\phi\beta_w+\alpha\right] \tag{31}$$

Substituting for the specific storage in the mass accumulation term yields:

$$\frac{\partial}{\partial t}(\phi\rho_w)=\rho_w S_s\frac{\partial h}{\partial t} \tag{32}$$

Substituting for the left-hand side in the Statement of Mass Conservation, Equation (24):

$$\rho_w S_s\frac{\partial h}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[\rho_w r K_r\frac{\partial h}{\partial r}\right]+\frac{\partial}{\partial z}\left[\rho_w K_z\frac{\partial h}{\partial z}\right] \tag{33}$$

The final form of governing equation for the transient flow is obtained by dividing through by the density of water, and assuming that the hydraulic conductivity is uniform:

$$S_s\frac{\partial h}{\partial t}=K_r\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial h}{\partial r}\right]+K_z\frac{\partial^2 h}{\partial z^2} \tag{34}$$

If the vertical hydraulic gradients are negligible, the governing equation reduces to:

$$S_s \frac{\partial h}{\partial t} = K_r \frac{1}{r} \frac{\partial}{\partial r}\left[r \frac{\partial h}{\partial r} \right] \quad (35)$$

This is the form of the governing equation that underlies most of the analytical approaches that are adopted in hydrogeology to interpret pumping tests. The governing equations for the transient radial flow of oil in the petroleum engineering formulation, Equation (17) is identical in form to the governing equation that hydrogeologists have traditionally taken as the starting point for most pumping test interpretation models, Equation (35).

4. ANALYTICAL SOLUTION FOR THE "LINE SINK" PROBLEM – THE THEIS SOLUTION

A classic solution of pressure test analysis is the "Line Sink" solution, also referred to as the "Exponential-integral" solution. In this section it is shown that the solution is identical to the Theis solution, a staple of hydrogeologic practice.

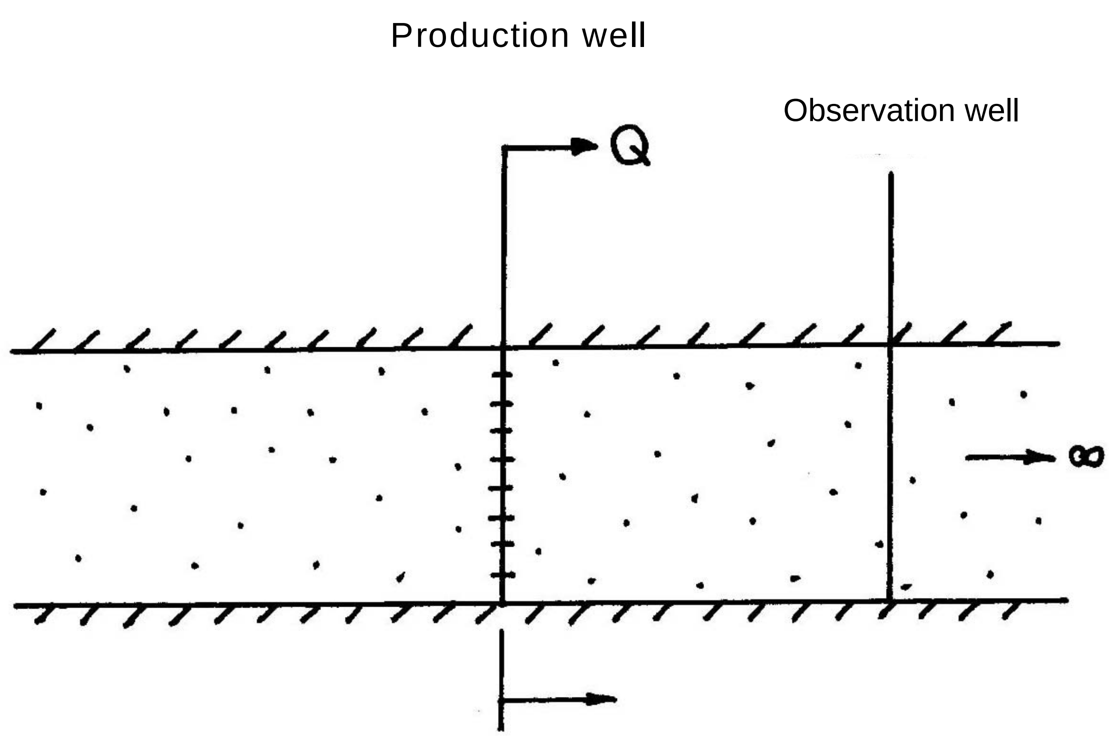


Figure 1. Conceptual model for the "Line Sink" solution.

The conceptual model for the "Line Sink" solution is shown schematically in Figure 1. The following key assumptions are invoked:

- A single well penetrates the full thickness of an areally extensive aquifer;
- The aquifer is bounded across its top and bottom surfaces by impermeable strata;
- The properties of the aquifer are uniform and isotropic;
- All water extracted by the pumping well is derived from storage; and
- The pumping well and observation wells have infinitesimal diameters (and are therefore approximated as lines).

4.1 Petroleum Engineering Formulation

The starting point for the analysis is Equation (17):

$$\frac{\phi\mu_w c_t}{k_r}\frac{\partial p}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial p}{\partial r}\right] \tag{17}$$

Invoking the assumption that the aquifer is areally extensive, the outer boundary condition is written as:

$$p\left(r \to \infty, t\right) = p_i \tag{36}$$

For simplicity, it is assumed here that the hydraulic head before the start of pumping, h_i, is uniform. However, as noted previously, this assumption is not necessary as the analysis is cast in terms of drawdown – that is, the changes in hydraulic head due only to pumping.

Invoking the assumption that the pumping well penetrates the full thickness of the aquifer and has infinitesimal diameter, the inner boundary condition is written as:

$$\lim_{r \to 0} -2\pi rh\frac{k_r}{\mu}\frac{\partial p}{\partial r} = -q \tag{37}$$

To be consistent with the notation of Earlougher (1977), the aquifer thickness is designated h and the pumping rate is designated q. The sign convention is that a positive value of q denotes extraction of fluid by the pumping well.

Again for simplicity, it is assumed that the pressure is uniform at the start of pumping and the initial conditions are written as:

$$p\left(r, 0\right) = p_i \tag{37}$$

The solution of Equation (17) subject to (35), (36) and (37) was derived first in the context of the analysis of the conduction of heat in solids (see for example Carslaw and Jaeger, 1959). Using the present notation, the solution is written as:

$$p(r,t) = p_i - \frac{q}{2\pi kh}\left[-\frac{1}{2}Ei\left(-\frac{\phi\mu c_t}{k}\frac{r^2}{4t}\right)\right] \qquad (38)$$

The term $Ei(-x)$ is the referred to as the Exponential Integral, and is defined as:

$$Ei(-x) = -\int_x^\infty \frac{1}{y} EXP\{-y\}\, dy \qquad (39)$$

The exponential integral is plotted in Figure 2.

The pressure change due to pumping is obtained by rearranging (38):

$$\Delta p(r,t) = p_i - p(r,t) = \frac{q}{2\pi kh}\left[-\frac{1}{2}Ei\left(-\frac{\phi\mu c_t}{k}\frac{r^2}{4t}\right)\right] \qquad (40)$$

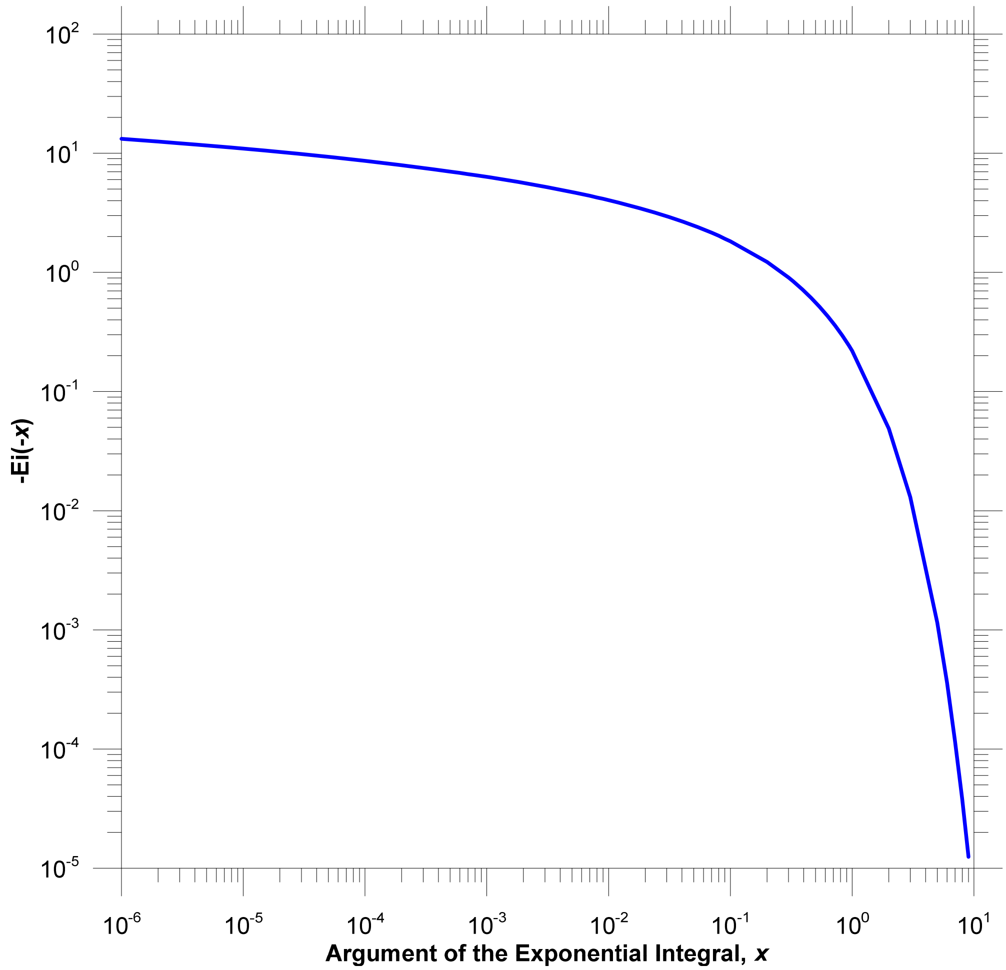


Figure 2. The Exponential Integral –Ei(-*x*).

4.2 Hydrogeology Formulation

The starting point for the analysis is Equation (35):

$$S_s \frac{\partial h}{\partial t} = K_r \frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial h}{\partial r} \right] \tag{35}$$

Invoking the assumption that the aquifer is areally extensive, the outer boundary condition is written as:

$$h\left(r \to \infty, t\right) = h_i \tag{41}$$

For simplicity, it is assumed here that the hydraulic head before the start of pumping, h_i, is uniform. However, as noted previously, this assumption is not necessary as the analysis is cast in terms of drawdown – that is, the changes in head due only to pumping.

Invoking the assumption that the pumping well penetrates the full thickness of the aquifer and has infinitesimal diameter, the inner boundary condition is written as:

$$\lim_{r \to 0} -2\pi rb\, K_r \frac{\partial h}{\partial r} = -Q \tag{42}$$

In the hydrogeologic development to avoid confusion the aquifer thickness is designated b, and to distinguish it from the Darcy flux the pumping rate is designated with Q. A positive value of Q is assumed for the withdrawal of groundwater at the pumping well.

Again for simplicity, it is assumed that the head is uniform at the start of pumping and the initial conditions are written as:

$$h\left(r, 0\right) = h_i \tag{43}$$

The solution of Equation (35) subject to (41), (42) and (43) was presented in Theis (1935):

$$s\left(r,t\right) = h_i - h\left(r,t\right) = \frac{Q}{4\pi K_r b} \int_u^\infty \frac{1}{y} EXP\left\{-y\right\} dy \tag{44}$$

where the argument of the integral is:

$$u = \frac{r^2 S_s}{4K_r t} \tag{45}$$

The integral in Equation (45) is the Generalized Exponential Integral of order $n = 1$, $E_1(x)$ (Abramowitz and Stegun, 1972):

$$E_1(x) = \int_x^{\infty} \frac{1}{y} EXP\{-y\}\, dy \tag{46}$$

To show how this integral is related to the Exponential Integral, $-Ei(-x)$ of Equation (39) requires returning to the definition of the Exponential Integral.

$$Ei(x) = -\int_{-x}^{\infty} \frac{1}{y} EXP\{-y\}\, dy \tag{47}$$

From this definition we obtain directly:

$$Ei(-x) = -\int_x^{\infty} \frac{1}{y} EXP\{-y\}\, dy \tag{48}$$

and

$$-Ei(-x) = \int_x^{\infty} \frac{1}{y} EXP\{-y\}\, dy \tag{49}$$

The right-hand side is identical to the definition of $E_1(x)$. Therefore, the solution for the hydrogeology formulation can be written as:

$$s(r,t) = \frac{Q}{4\pi K_r b}\left[-Ei\left(-\frac{r^2 S_s}{4K_r t}\right)\right] \tag{50}$$

This solution is traditionally written as:

$$s(r,t) = \frac{Q}{4\pi K_r b} W(u) \tag{51}$$

with $W(u)$ referred to as the Theis well function, and Equation (51) is referred to as the Theis solution. The argument of the Theis well function is the dimensionless quantity u defined in Equation (45). The Theis well function is plotted in Figure 3.

Recognizing the equivalence between the Exponential Integral and the Theis well function, the "Line Sink" solution, Equation (40), and the Theis solution, Equation (51), are <u>identical</u> in form.

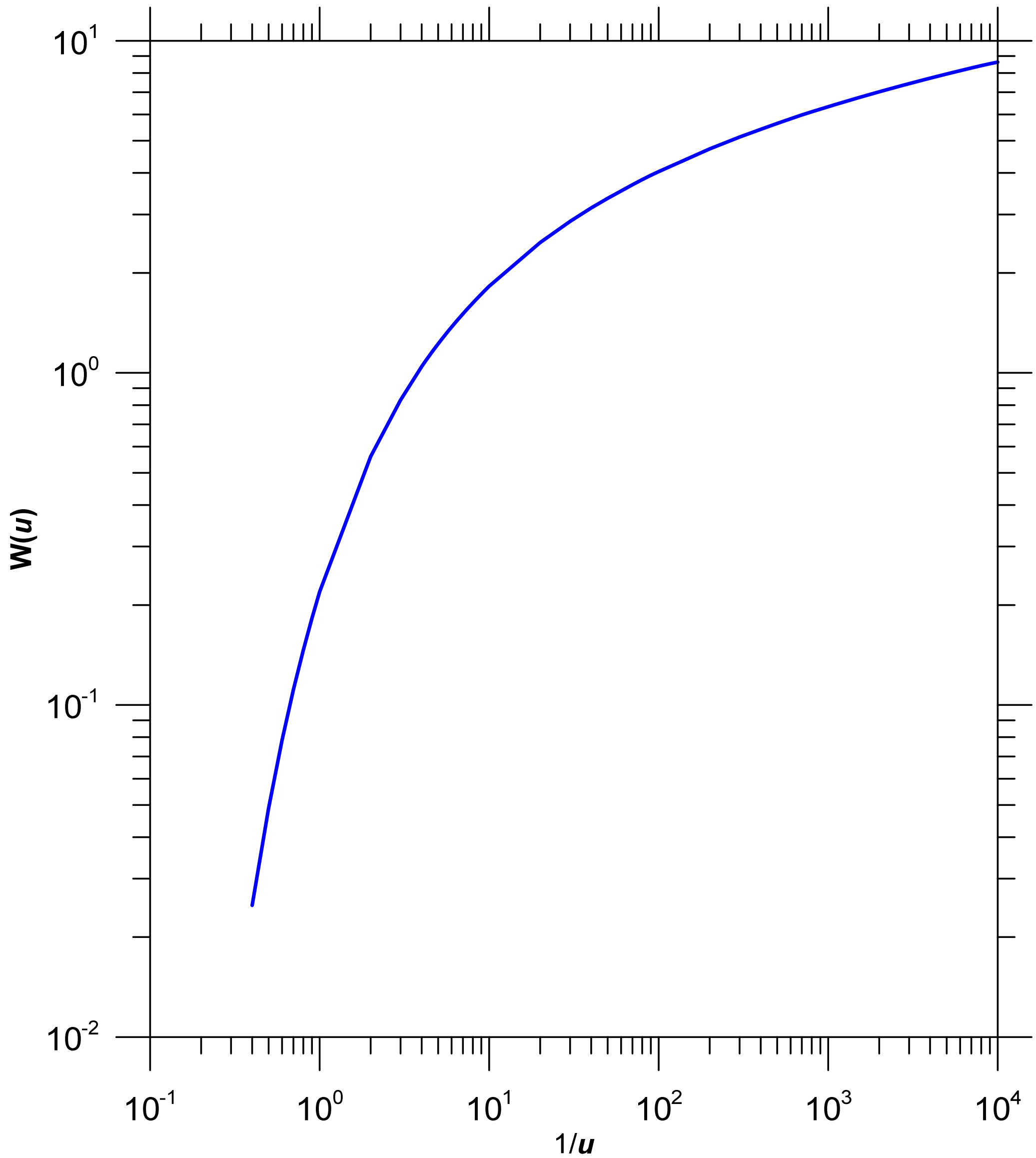


Figure 3. The Theis well function.

5. ANALYSIS OF THE RECOVERY FOLLOWING PUMPING FROM A "LINE SINK": THE HORNER/COOPER-JACOB ANALYSIS

5.1 Petroleum Engineering Formulation

The starting point for the analysis is the "Line Sink" solution, Equation (38):

$$p(r,t) = p_i - \frac{q}{2\pi kh}\left[-\frac{1}{2}Ei\left(-\frac{\phi\mu c_t}{k}\frac{r^2}{4t}\right)\right] \tag{38}$$

Horner (1951) considered the case of pumping at a constant rate followed by the cessation of pumping, referred to in the petroleum engineering literature as the *shutting-in* of the well. The subsequent increase in pressure back to the initial pressure pi is referred to as *build-up*.

The boundary value problem is linear; therefore, the principle of superposition can be applied to derive the solution for the pressure change for time-varying pumping. If the duration of pumping is designated *t*, and the elapsed time since the end of pumping is designated Δ*t*, the pressure during the build-up period for an ideal "Line Sink" is given by:

$$p(r,t) = p_i - \frac{q}{2\pi kh}\left[-\frac{1}{2}Ei\left(-\frac{\phi\mu c_t}{k}\frac{r^2}{4(t+\Delta t)}\right)\right] + \frac{q}{2\pi kh}\left[-\frac{1}{2}Ei\left(-\frac{\phi\mu c_t}{k}\frac{r^2}{4\Delta t}\right)\right] \tag{52}$$

Horner indicated that for values of the argument of the Exponential Integral less than about 0.01, the Exponential Integral could be approximated closely by:

$$Ei(-x) \approx 0.5772 + \ln\{x\} \tag{53}$$

Assuming that the limits of applicability of (53) are satisfied, Equation (52) can be approximated as:

$$p(r,t) = p_i - \frac{q}{2\pi kh}\left[-\frac{1}{2}\left[0.5772 + \ln\left\{\frac{\phi\mu c_t}{k}\frac{r^2}{4(t+\Delta t)}\right\}\right]\right] + \frac{q}{2\pi kh}\left[-\frac{1}{2}\left[0.5772 + \ln\left\{\frac{\phi\mu c_t}{k}\frac{r^2}{4\Delta t}\right\}\right]\right] \tag{54}$$

Simplifying Equation (54) yields:

$$p(r,t) = p_i + \frac{q}{4\pi kh}\left[\ln\left\{\frac{\phi\mu c_t}{k}\frac{r^2}{4(t+\Delta t)}\right\}\right] - \frac{q}{4\pi kh}\left[\ln\left\{\frac{\phi\mu c_t}{k}\frac{r^2}{4\Delta t}\right\}\right]$$

Making use of the properties of the log function and simplifying:

$$p(r,t) = p_i - \frac{q}{4\pi kh}\ln\left\{\frac{t+\Delta t}{\Delta t}\right\} \tag{55}$$

Example analysis

The data from well CB-161 from an example analysis in Horner's original paper are plotted in Figure 4.

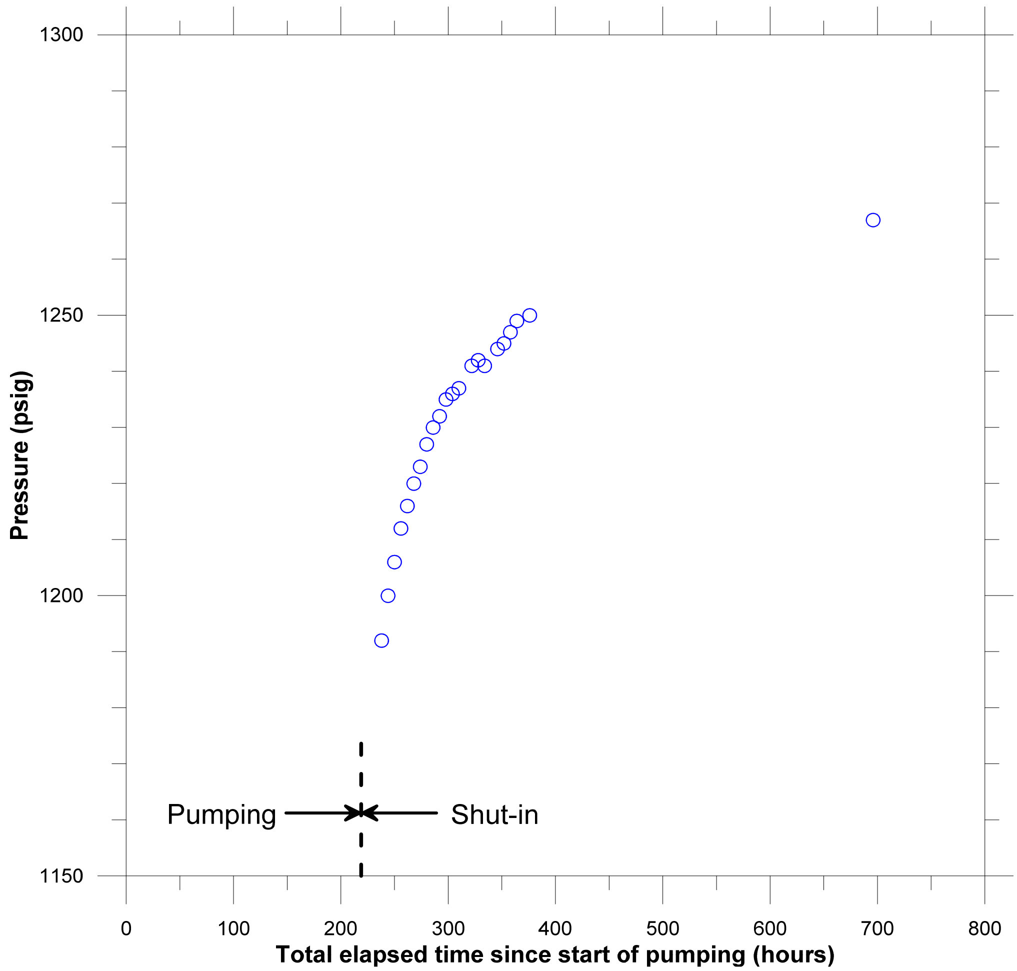


Figure 4. Data from example Horner recovery analysis (data from Horner, 1951).

A plot of the Horner analysis is shown in Figure 5. In the analysis, the pressure in the test interval recorded after pumping has stopped is plotted against the logarithm of $(t+\Delta t)/\Delta t$, following the form of Equation (55). As shown in the figure, Horner developed to the analysis to estimate two quantities, the initial pressure that would have been observed before the start of pumping, and the *permeability-thickness product*, *kh*.

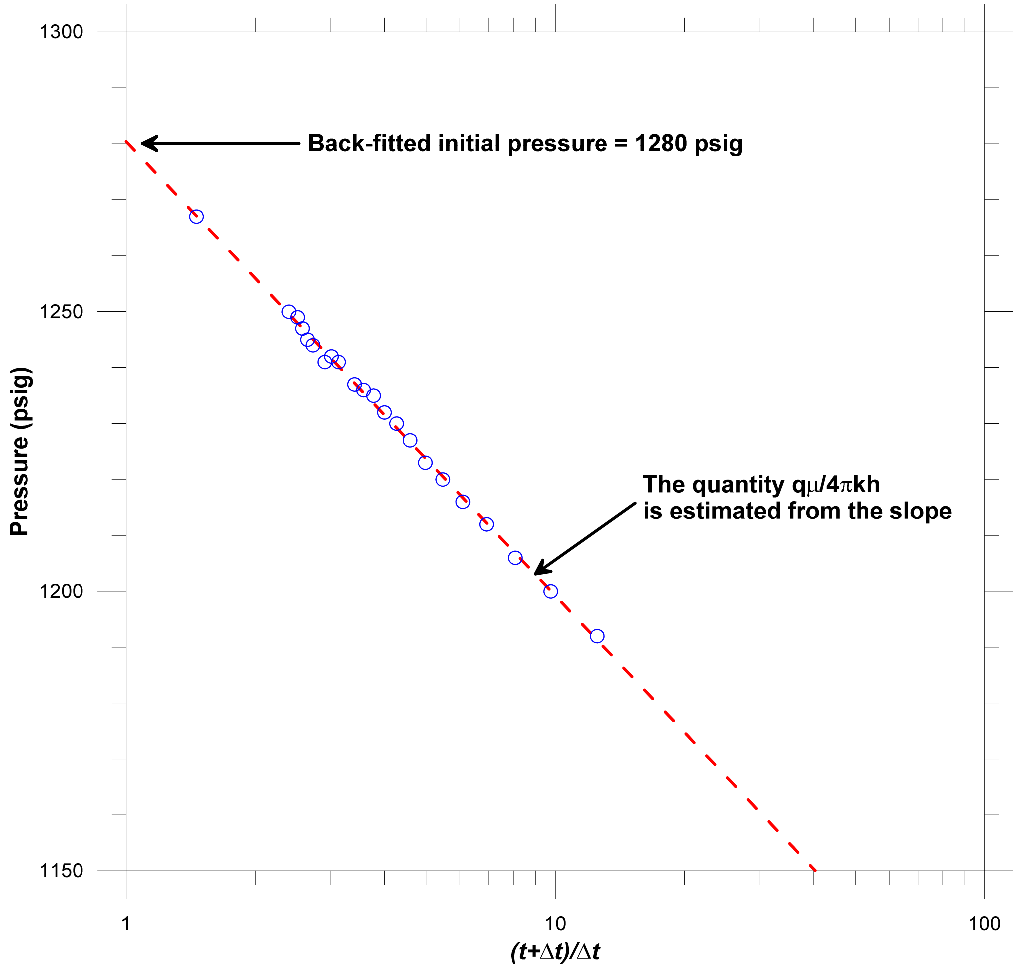


Figure 5. Horner analysis to estimate initial pressure p_i.

5.2 Hydrogeology Formulation

The hydrogeology formulation of the Horner recovery analysis is referred to as the *Cooper-Jacob straight-line recovery analysis*, and since its introduction in 1946 has become a standard technique in pumping test analysis (Cooper and Jacob, 1946).

The starting point for the Cooper-Jacob straight-line analysis is the Theis solution, Equation (51):

$$s(r,t) = \frac{Q}{4\pi K_r b} W(u) \tag{51}$$

If the total elapsed time since the start of pumping is designated t, and the duration of pumping is designated t_{pump}, the elapsed time since the end of pumping is calculated as:

$$t' = t - t_{pump} \tag{56}$$

The drawdown after the end of pumping is given by:

$$s(r,t) = \frac{Q}{4\pi K_r b} W(u) - \frac{Q}{4\pi K_r b} W(u') \tag{57}$$

The arguments of the Theis well function are:

$$u = \frac{r^2 S_s}{4K_r t} \qquad u' = \frac{r^2 S_s}{4K_r t'} \tag{58}$$

Approximating the Theis well function with its first two terms, as was done with the Horner analysis and the Exponential Integral:

$$W(x) \approx -0.5772 - \ln\{x\} \tag{59}$$

Assuming that the limits of applicability of (59) are satisfied, Equation (57) can be approximated as:

$$s(r,t) = \frac{Q}{4\pi K_r b}\left[-0.5772 - \ln\{u\}\right] - \frac{Q}{4\pi K_r b}\left[-0.5772 - \ln\{u'\}\right] \tag{60}$$

Simplifying Equation (60) yields:

$$s(r,t) = -\frac{Q}{4\pi K_r b}\ln\{u\} + \frac{Q}{4\pi K_r b}\ln\{u'\} \tag{61}$$

Making use of the properties of the log function and simplifying:

$$s(r,t) = \frac{Q}{4\pi K_r b}\ln\left\{\frac{u'}{u}\right\} \tag{62}$$

Substituting back for u' and u and simplifying yields the final form of the solution:

$$s(r,t) = \frac{Q}{4\pi K_r b}\ln\left\{\frac{t}{t'}\right\} \tag{63}$$

Equation (63) is identical in form to the Horner approximation, recognizing the following equivalence indicated below.

Parameter	**Horner approximation**	**Cooper-Jacob approximation**
Total elapsed time	$t+\Delta t$	t
Time since end of pumping	Δt	t'

6. SUMMARY

These materials have been prepared to show through a detailed development how the foundations of pressure test analysis for petroleum engineering relate to the foundations of hydrogeologic pumping test interpretation. It has been shown that:

- The governing equations for the transient radial flow of oil in the petroleum engineering formulation is identical in form to the governing equation that hydrogeologists use as their starting point for most pumping test interpretation models;

- The foundational models for the interpretation of pumping tests in ideal settings, the "Line Sink" solution in petroleum engineering, and the Theis solution in hydrogeology are identical in form; and

- The foundation model for the interpretation of the observations collected after the end of pumping in ideal settings, the Horner approximation in petroleum engineering, and the Cooper-Jacob straight-line recovery analysis are identical in form.

By studying these developments, hydrogeologists should be better equipped to take advantage of the wealth of literature and analysis techniques presented in the petroleum engineering literature. A hint of that wealth is presented in the final section of selected references.

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Appendix 4

Case Study: Pumping Test in a Buried-Valley Aquifer System, Estevan, Saskatchewan

Overview

The recommended approaches for interpreting pumping tests are illustrated with the detailed analyses of a well-documented pumping test conducted in a buried-valley aquifer near the town of Estevan in southern Saskatchewan. Buried-valley aquifers are a major element of the hydrogeology of the Western Glaciated Plains (Lennox and others, 1988) and are particularly important in the area of the Athabasca Oil Sands (Hackbarth and Nastasa, 1979; Andriashek and Atkinson, 2007; Atkinson and Slattery, 2011). The Estevan test has been selected because it is well-documented and because the aquifer in which it was conducted has been the focus of ongoing studies that extend over a fifty-year period (Walton, 1970, Maathuis and van der Kamp, 2003; van der Kamp and Maathuis, 2012). Because of this long record of study, the Estevan pumping test also provides an excellent opportunity to review the application of the methods used to estimate the sustainable yield of a production well. The results of the analyses and discussion reveal that the estimation of the sustainable yield of a well is a subtle task, and that a wide range of results may be obtained for a buried-valley system typical to the Canadian Prairies.

1. Hydrogeologic Setting

In March 1965, the Saskatchewan Research Council conducted a constant-rate pumping test about 13 miles northwest of Estevan, Saskatchewan. The test was conducted in a long, sinuous paleochannel infilled with permeable sand and gravel. The channel is part of a complex network of buried valley aquifers across the Canadian Prairies of Western Canada. A recent interpretation of this network is reproduced in Figure 1 (Cummings and others, 2011).

Descriptions of the hydrogeology of the Estevan area and the responses to pumping are presented in van der Kamp and Maathuis (2002), Maathuis and van der Kamp (2003) and van der Kamp and Maathuis (2012). The current interpretation of the buried channel aquifer network in the Estevan area is shown in Figure 2. Similar large–scale networks have been mapped throughout Alberta (Farvolden, 1963; Evans and Campbell, 1995; Andriashek and Atkins, 2007; Rayner and Rosenthal, 2008; and Atkinson and Slattery, 2011). A geologic log for the production well is reproduced in Figure 3. The buried valley aquifer is overlain by about 150 m of low-permeability glacial till.

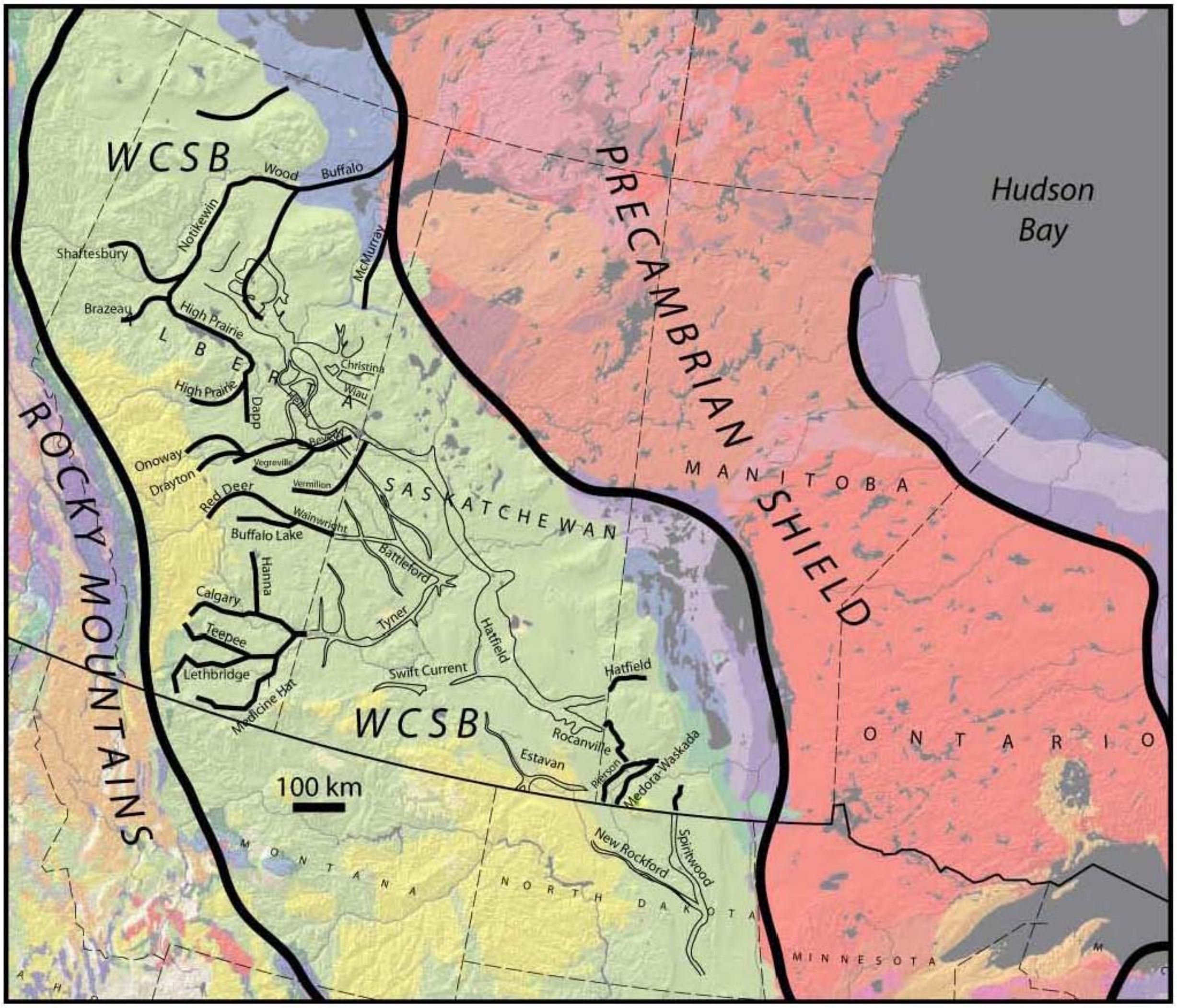

Figure 1. Buried channel aquifers in Western Canada.
(Reproduced from Cummings and others, 2011)

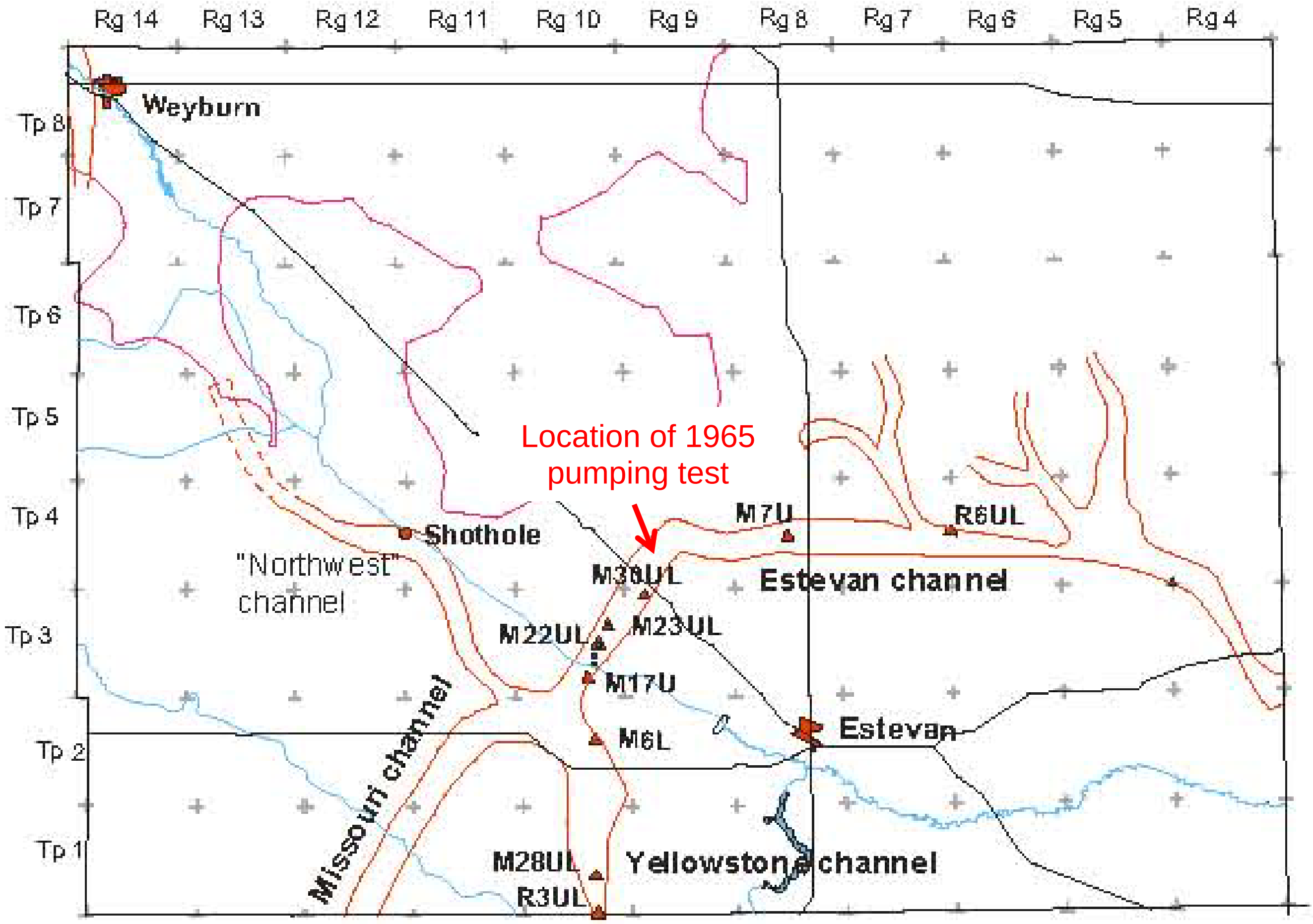

Figure 2. Buried valley aquifer system in southern Saskatchewan.
(Reproduced from Maathuis and van der Kamp (2003))

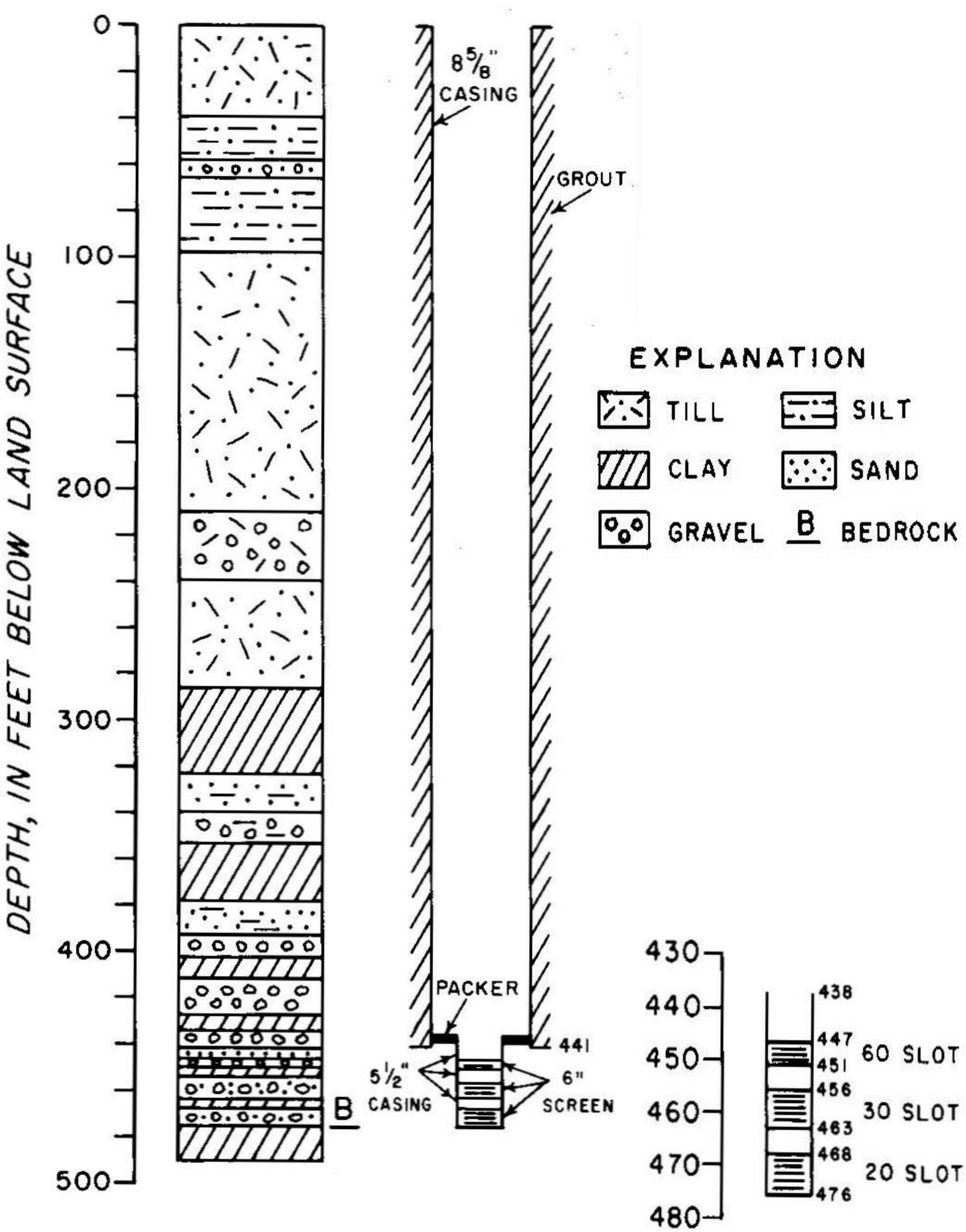

Figure 3. Geologic log and production well construction details.
Adapted from Walton (1970)

2. Summary of Pumping Test

Pumping started on March 4, 1965 at 3:00 PM and continued until March 12 at 2:00 PM. The duration of pumping was 11,520 minutes. The pumping rate was held constant by means of a gate valve installed in the discharge pipe. A circular orifice and a manometer tube installed in the end of the discharge pipe were used to measure the rate of pumping. The rate of pumping varied between 457 igpm (imperial gallons per minute) and 464 igpm, with an average pumping rate of 460 igpm (3,010 m^3/d).

Water levels were measured at the production well and at three observation wells. The locations of the wells are shown in Figure 4.

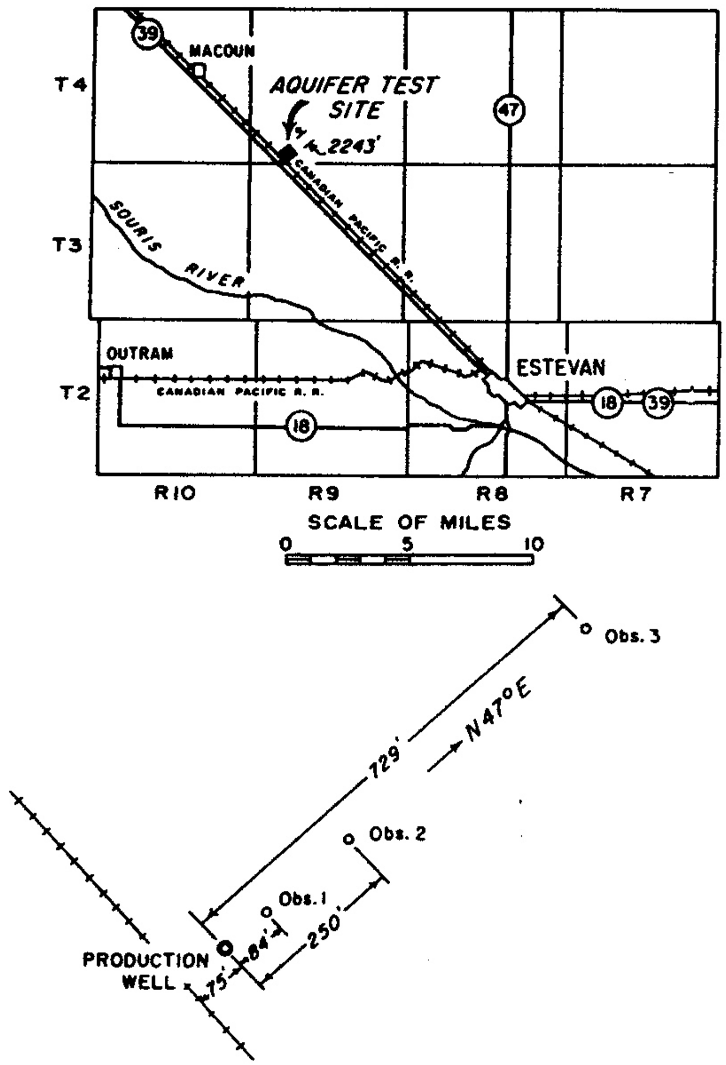

Figure 4. Locations of wells monitored during the Estevan pumping test.
Adapted from Walton (1970)

3. Preliminary Estimate of Aquifer Transmissivity

A step test was conducted in anticipation of the Estevan constant-rate pumping test. The results of the step test are provided in the original report of the test (Walton, 1965). The drawdowns observed during the three steps of the Estevan step test are plotted in Figure 5.

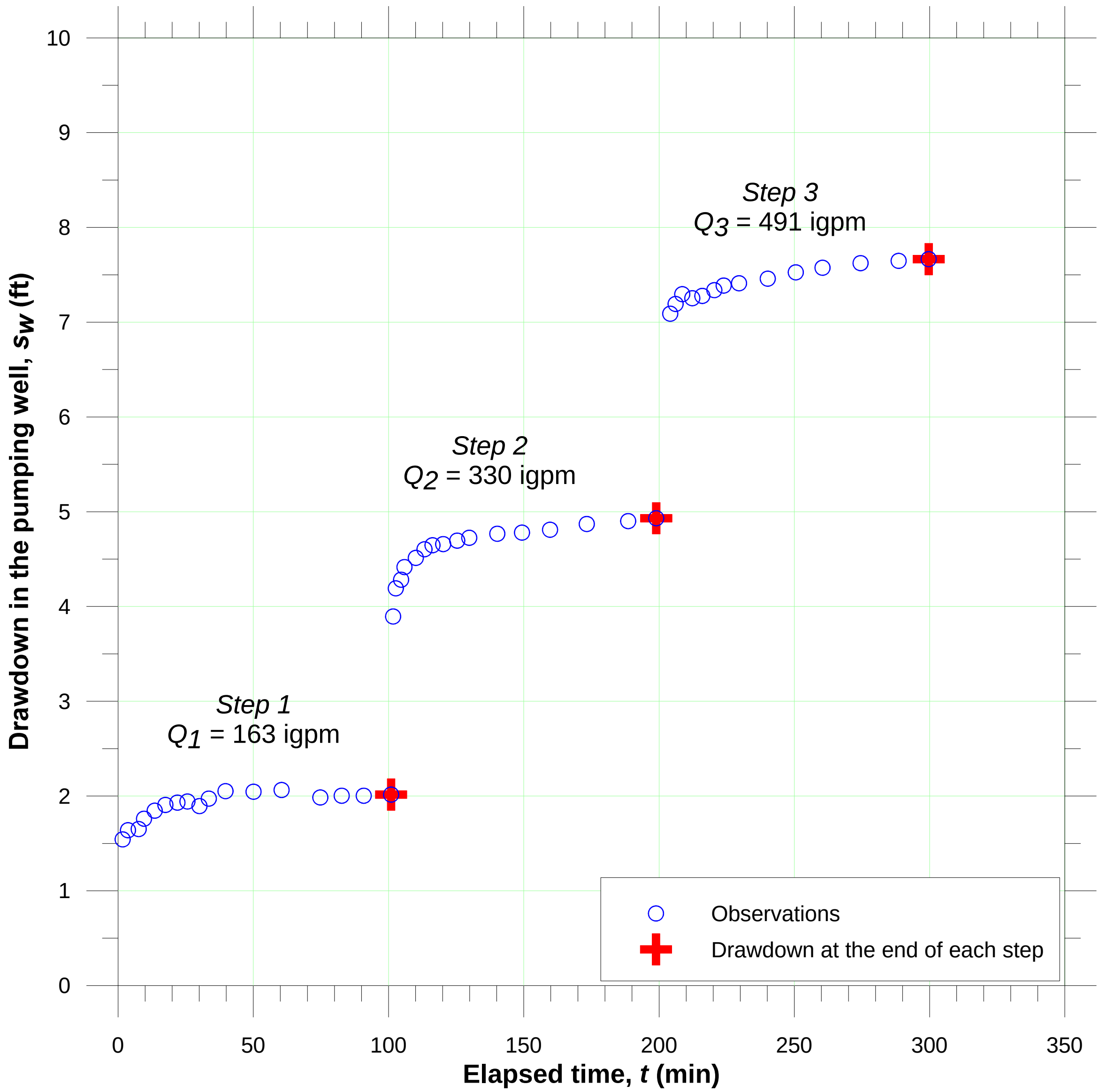

Figure 5. Estevan step test data.
Data from Walton (1965)

Values of the specific drawdown, s_w/Q, defined as the drawdown in the pumping well at the end of each step divided by the pumping rate during that step, are plotted in Figure 6. As shown in the figure, the specific drawdown does not vary significantly with the pumping rate. This suggests that nonlinear well losses are not significant during this test. The drawdowns at the ends of the last two steps are approximated closely with the relation:

$$s_w = 0.015\,Q$$

where s_w are specified in feet and the pumping rate Q is specified in igpm.

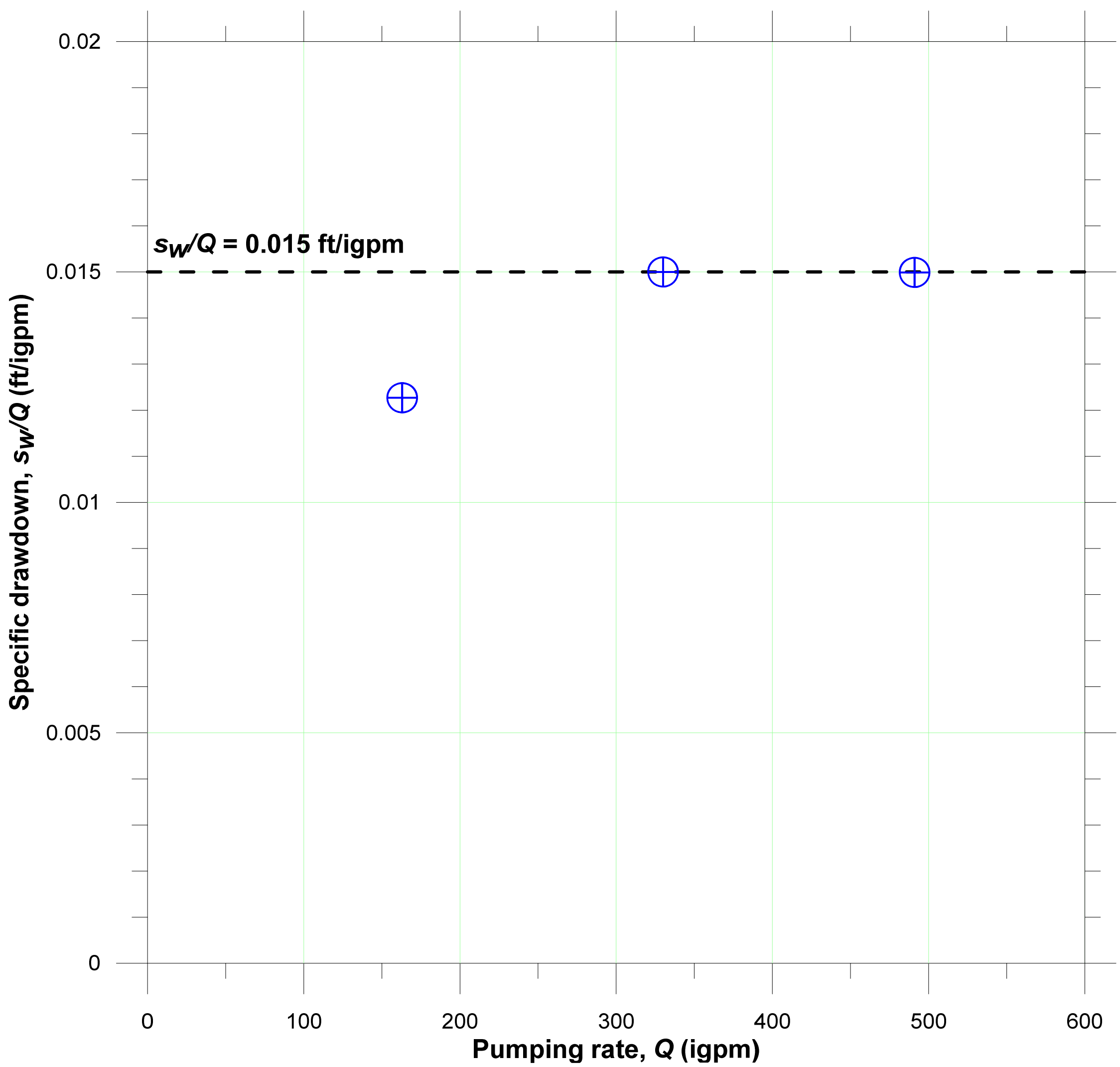

Figure 6. Estevan step test, Hantush-Bierschenk plot.

As a check on the Hantush-Bierschenk analysis, the pumping rate is plotted against the drawdown at the end of each step in Figure 7. This plot is referred to as a *specific capacity plot*. The data approximate closely a linear relation, confirming that the nonlinear well losses are likely not significant.

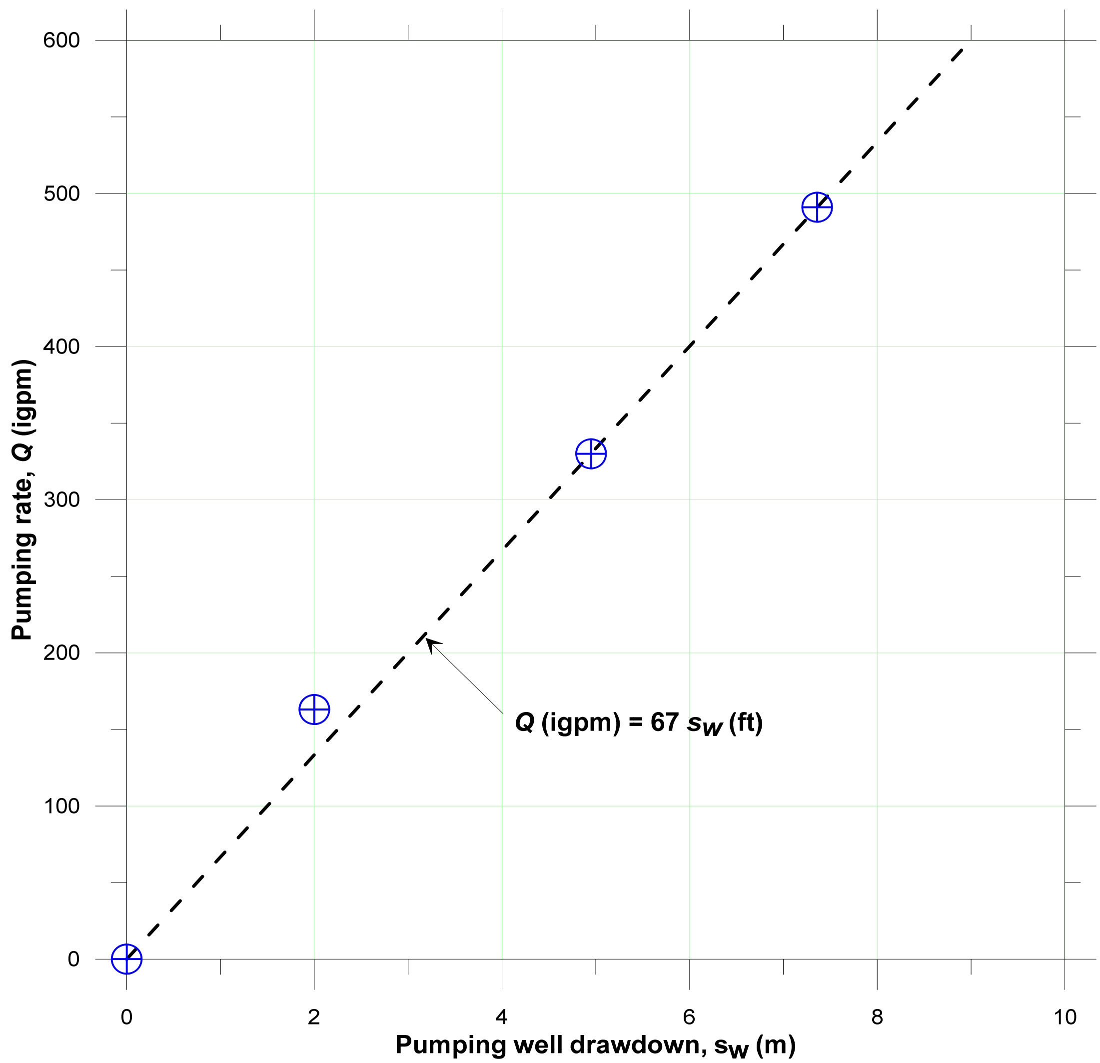

Figure 7. Specific capacity plot of Estevan step test.

The *y*-intercept of Hantush-Bierschenk plot corresponds to the coefficient *B* of the Jacob (1947) model. The reciprocal of *B* can be interpreted as the specific capacity with the nonlinear well losses removed. Using this value and the approach of Driscoll (1986), a first-cut transmissivity estimate is calculated as:

$$T \approx 1.3 \times \frac{1}{B}$$
$$= 1.3 \left|\frac{1}{0.015\ ft/igpm}\right| \left|\frac{ft^3}{6.229\ igal}\right| \left|\frac{1440\ min}{day}\right| = 20{,}040\ ft^3/d$$

A more refined, but still preliminary, estimate of the transmissivity can be developed by matching the complete record of drawdowns during the step test with the Theis solution generalized for time-varying pumping:

$$s_w(t) = \sum_{i=1}^{NS} \frac{\Delta Q_i}{4\pi T} W\left(\frac{r_w^2 S}{4T(t - ts_i)}\right)$$

Here NS denotes the number of steps in the test, ΔQ_i denotes the pumping increment, t is the total elapsed time since the start of pumping, ts_i is the starting time for each step, r_w is the radius of the well, and T and S denote the transmissivity and storage coefficient, respectively.

The results of the analysis are shown in Figure 8. A relatively close match to the observations is achieved with the following parameters:

T = 23,300 ft^2/d; and
$S = 2\times10^{-4}$.

This estimate of the transmissivity is close to the first-cut estimate derived from the specific capacity calculation.

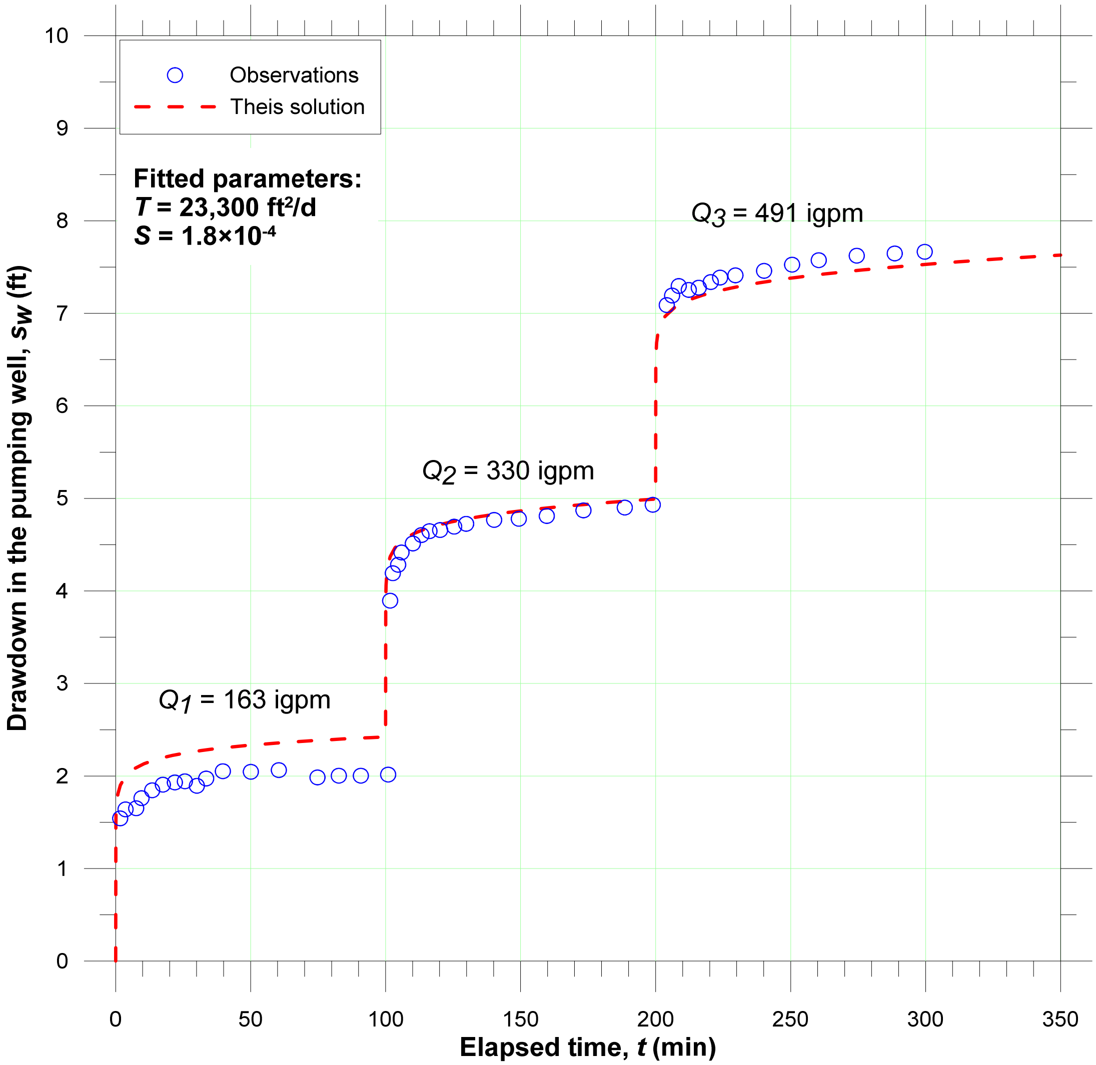

Figure 8. Match of the Estevan step drawdown data.

4. Diagnosis of Aquifer Characteristics

The drawdown data from the pumping well and the observation wells are plotted in three formats to assist in inferring the appropriate conceptual model for the aquifer:

- Log-log composite plot;
- Semi-log Drawdown Derivative plot; and
- Semi-log composite plot.

Diagnostic plot #1

The drawdowns for the pumping well and the three observation wells are plotted against time on log-log axes in Figure 9. As shown in the figure, the final portions of the drawdown records for each well approximate straight lines. This response is characteristic of a linear flow regime that is observed when a strip aquifer is pumped (see for example, Boonstra and Boehmer, 1986; Butler and Liu, 1991). This model is representative of a buried-valley aquifer for which the inflow across the valley walls and the leakage from the overlying confining unit are not significant over the duration of the pumping test.

Diagnostic plot #2

The Drawdown Derivatives for the pumping well and the three observation wells are plotted against time on semi-log axes in Figure 10. The Drawdown Derivatives are smoothed slightly with respect to the "raw" values calculated with the nearest-neighbor approach. Two distinct regimes are evident in the figure. Between about 10 and 50 minutes, the Drawdown Derivatives approach a plateau. This is designated as the period of Infinite-Acting-Radial-Flow (IARF). During this period, the drawdowns approximate the response that would be observed in an ideal confined aquifer of infinite extent. Beyond 50 minutes of pumping, the Drawdown Derivatives accelerate rapidly. This response is characteristic of an aquifer in which boundary effects are increasingly significant, which is again consistent with the conceptual model of buried-valley aquifer.

Diagnostic plot #3

A third diagnostic plot is presented in Figure 11. The drawdowns for the pumping well and the three observation wells are plotted against t/r^2 on semi-log axes, where r is the distance between the pumping well and each observation well. Cooper and Jacob (1946) refer to this as a composite plot. Two distinct regimes are evident for each well. For relatively small values of t/r^2, the drawdowns approximate a common straight line. The interval of this response corresponds to the period of Infinite-Acting-Radial-Flow for each well. For larger values of t/r^2, the drawdowns for each well appear to deviate systematically from the common straight line. The onset of this deviation marks the time at which the influence of pumping propagates to the boundary.

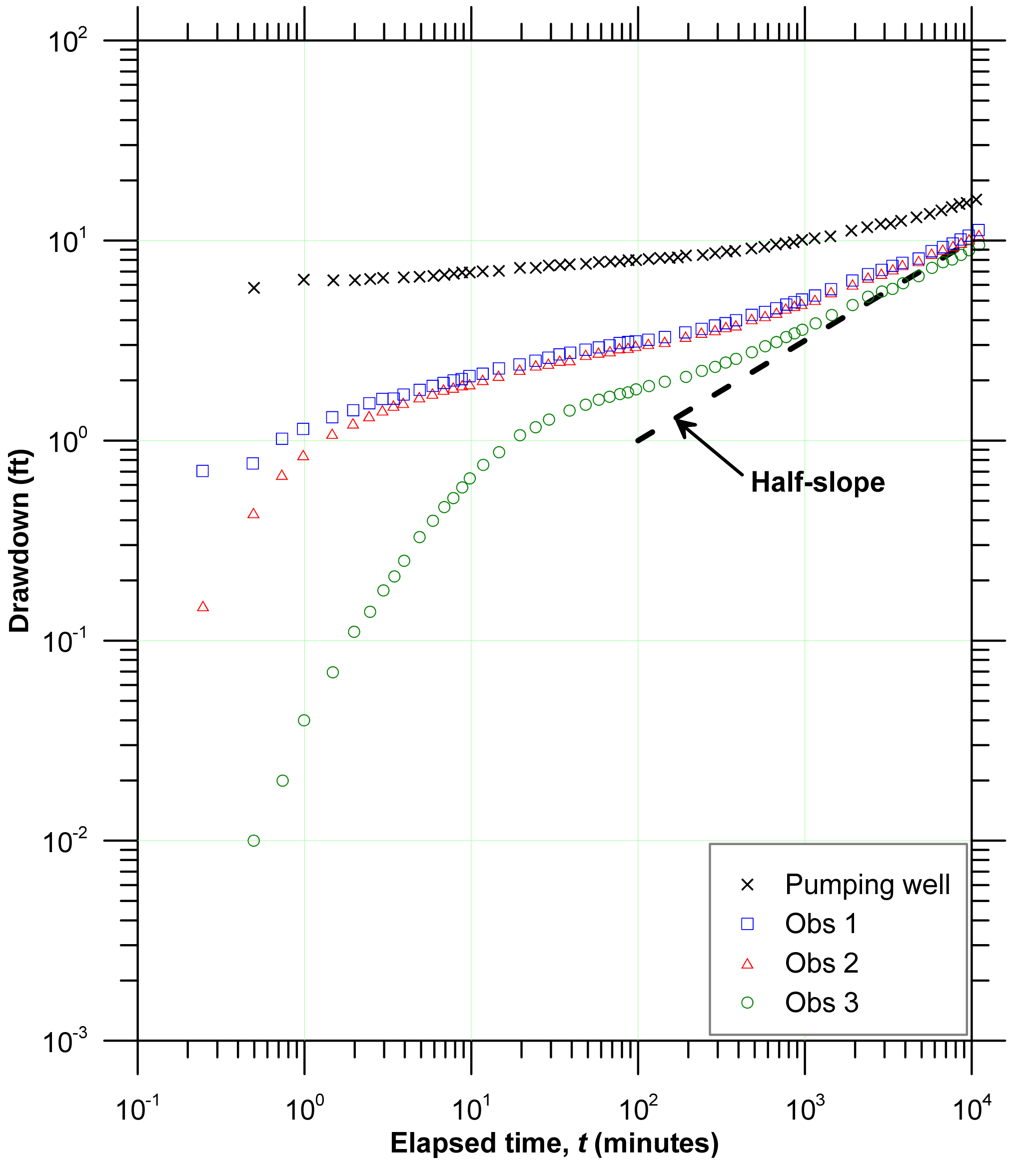

Figure 9. Log-log plot of drawdowns during the Estevan constant-rate pumping test.

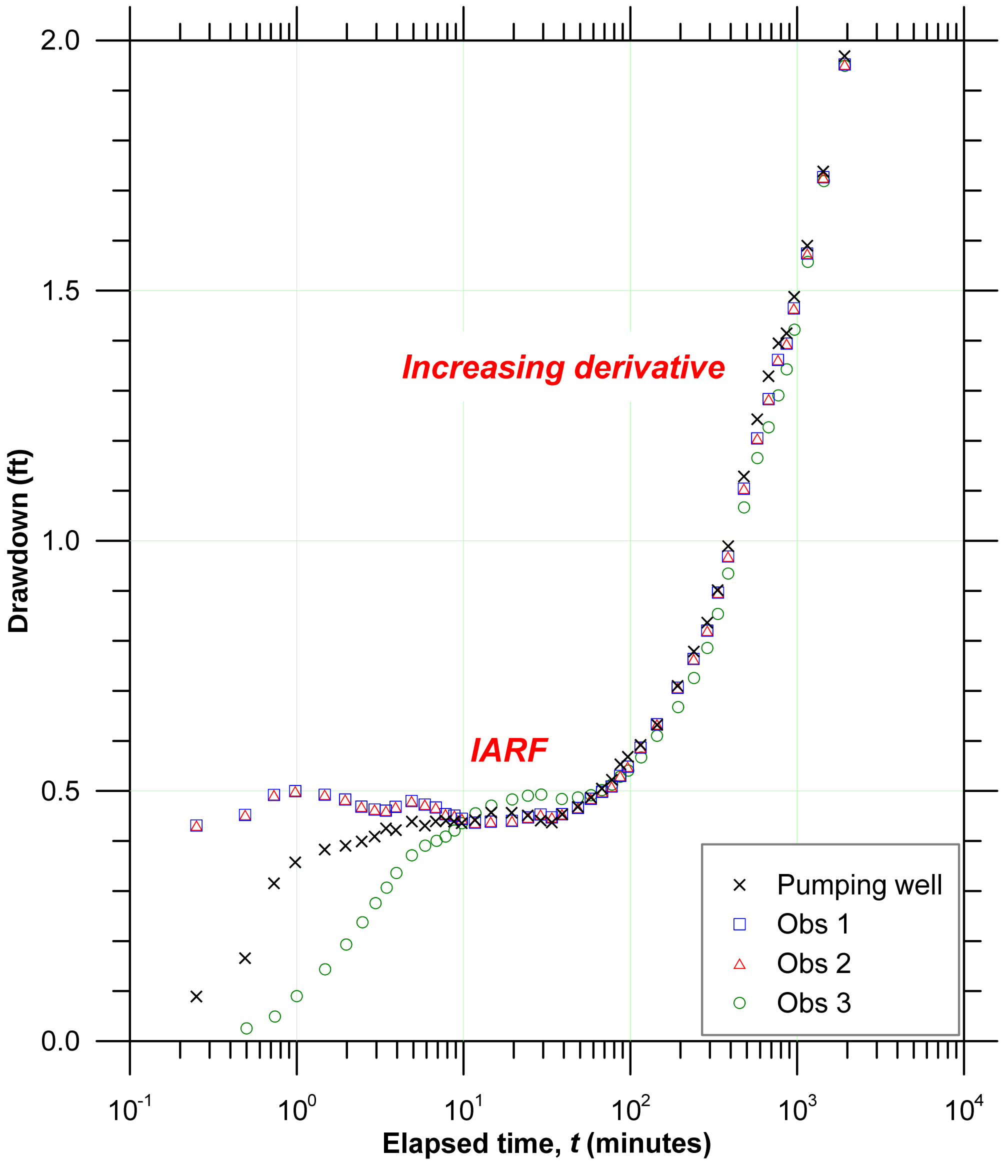

Figure 10. Semi-log plot of drawdown derivative.

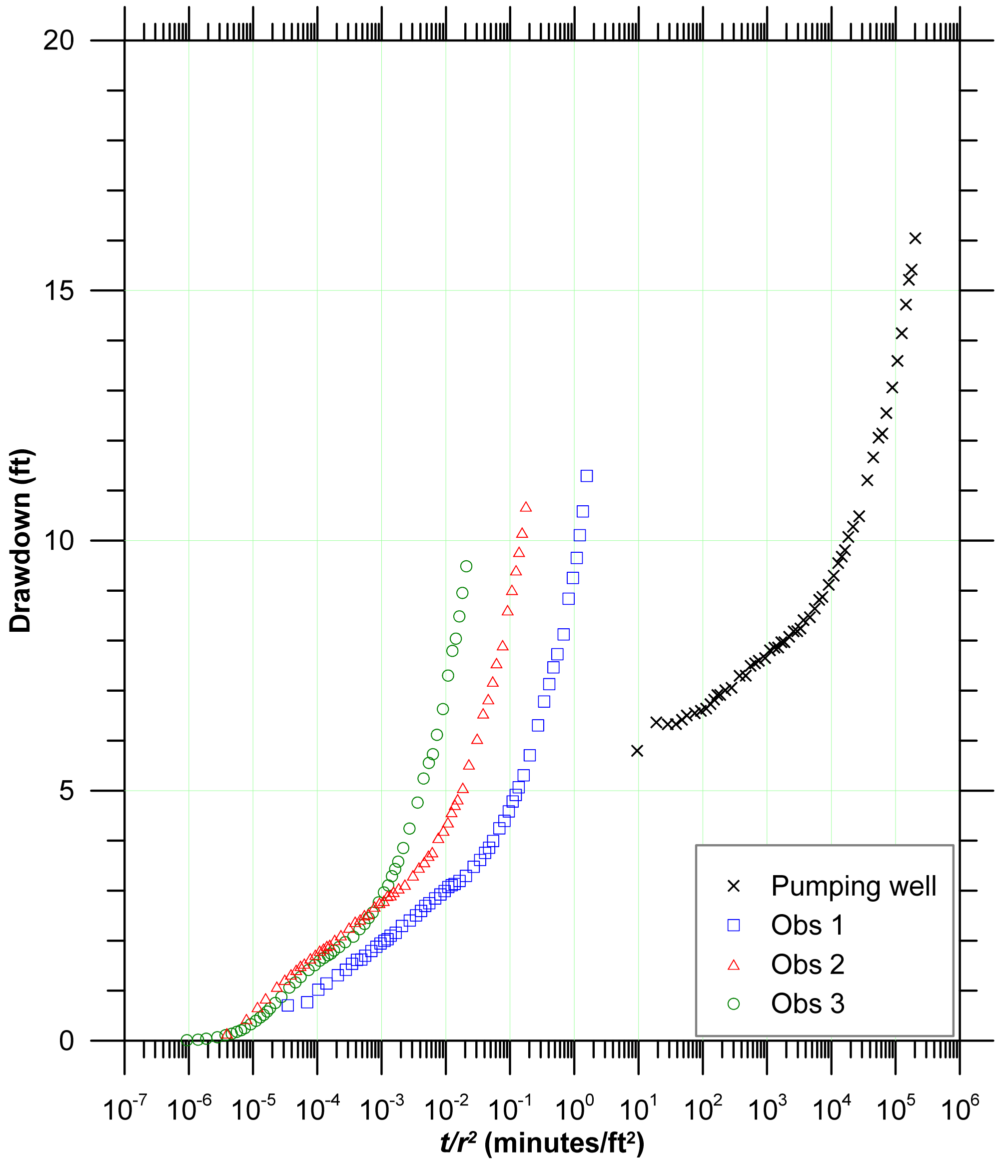

Figure 11. Semi-log composite plot of drawdowns.

5. Estimation of the Aquifer Properties

The inferences drawn from the three diagnostic plots provide key information to guide a refined interpretation of the pumping test:

- The Drawdown Derivate plot confirms that a portion of the response approximates an ideal confined aquifer;
- The composite plot of the drawdowns directs the analyst to the appropriate portion of the data to match with the Cooper-Jacob model; and
- The log-log plot of the drawdowns confirms that the appropriate conceptual model for a complete analysis is a buried-valley aquifer for which the inflow across the valley walls and the leakage from the overlying confining unit is not significant over the duration of the pumping test.

5.1 Cooper-Jacob straight-line analysis

The results of the Cooper-Jacob straight-line analysis are shown in Figure 12. The drawdowns do not all fall on the same straight line, which suggest that the aquifer is heterogeneous. However, the early-time slopes of the linear portions of the drawdowns are consistent, including the drawdowns for the pumping well. The straight-line portions of the responses therefore yield a single, internally consistent estimate of the transmissivity for all four wells. This consistency is a necessary condition for the reliability of the analysis, as the key assumption of the Cooper-Jacob model is that the bulk average transmissivity of the aquifer is constant.

The transmissivity is calculated from:

$$T = 2.303\,\frac{Q}{4\pi}\,\frac{1}{\Delta s}$$

where Q is the pumping rate, and Δs is the slope of the straight-line portion of the drawdown.
The common slope of the early-time response is about 0.92 ft per log cycle t/r^2. For a pumping rate of 460 igpm, the transmissivity is estimated as:

$$T = 2.303\,\frac{(460\ ipgm)}{4\pi}\,\frac{1}{(0.92\ ft)}\left|\frac{ft^3}{6.229\ gal}\right|\left|\frac{1440\ min}{day}\right| = 21{,}200\ ft^2/d$$

Using the straight line fit through the pumping well and Obs 1 drawdowns, a storage coefficient, S, of 2×10^{-4} is estimated. The storage coefficient is within the typical range for confined aquifers, 5×10^{-5} to 5×10^{-3} (Boonstra, 1989).

The results from the Cooper-Jacob analysis are consistent with the previous analysis of the step test. This consistency does <u>not</u> prove that the analyses are correct, but it is a necessary condition for a reliable interpretation.

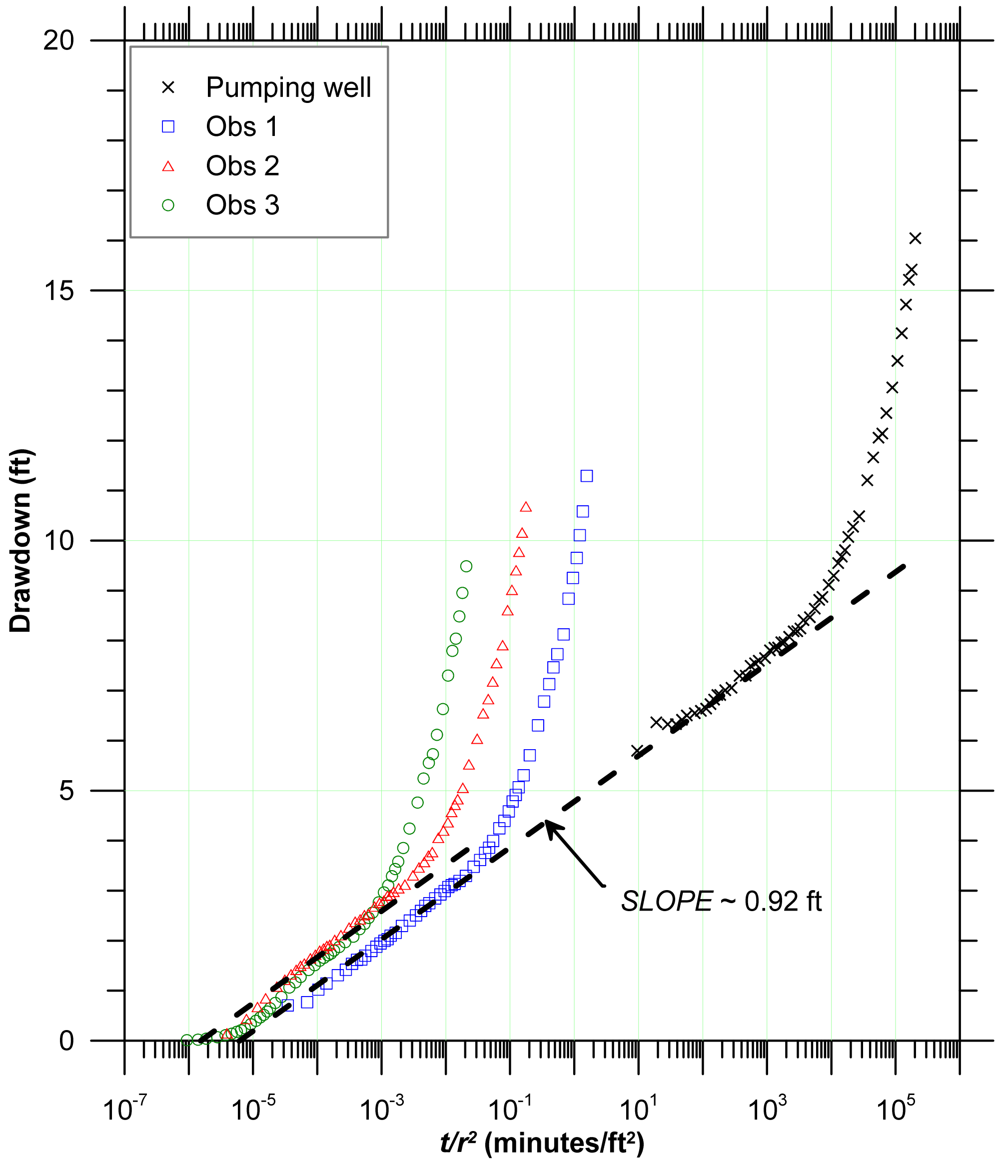

Figure 12. Cooper-Jacob straight-line analysis.

5.2 Buried-valley aquifer analysis

The drawdown data are interpreted with a buried valley aquifer analysis by retaining the conceptual model of an ideal confined aquifer, but assuming that the pumping well and observation wells are located along the axis of a long rectangular aquifer, bounded by parallel impermeable surfaces that penetrate the full thickness of the aquifer. The analysis is conducted with an automated implementation of image theory. The image well model is illustrated in Figure 13. The black circle indicates the real well and the white circles indicate the image wells, all of which pump at the same rate as the real well.

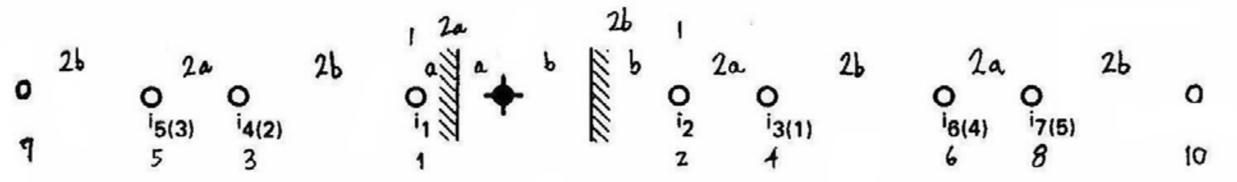

Figure 13. Image well model for a buried-valley aquifer.

The solution for pumping between two linear impermeable boundaries is given by (Kruseman and de Ridder, 1990; p. 114):

$$s(r,t) = \frac{Q}{4\pi T}\left[W\left(\frac{r^2 S}{4Tt}\right) + \sum_{i=1}^{\infty} W\left(A_{ri}^2 \frac{r^2 S}{4Tt}\right)\right]$$

where the right-hand term in brackets represent the contributions of the image wells. The quantity A_{ri} is defined as:

$$A_{ri} = \frac{r_i}{r}$$

with r the distance between the real well and the observation well, and r_i is the distance between the image well i and the observation well.

In theory, infinitely many image wells are required. In practice, the calculations frequently converge for a relatively small number. The number of image wells required depends on the location of the observation well and the elapsed time; a convergence analysis is generally required, in which the number of image wells is increased until the addition of another image well has negligible effect on the calculated drawdowns.

The solid lines in Figure 14 are calculated with a transmissivity of 19,800 ft^2/d, storage coefficient of 2.8×10^{-4}, with a specified valley width of 8,000 ft (2,430 m). The valley width is estimated through trial-and-error. As shown in the figure, it is possible to obtain a close match to the drawdowns from the pumping well and all three observation wells, with a consistent set of aquifer properties. The parameters are close to those estimated with the Cooper-Jacob straight-line analysis (T = 21,200 ft^2/d, S = 2.8×10^{-4}). As a further check on the analysis, in Figure 15 the Drawdown Derivatives for the analytical results plotted in Figure 14 are superimposed on the values calculated from the observations. The match is excellent.

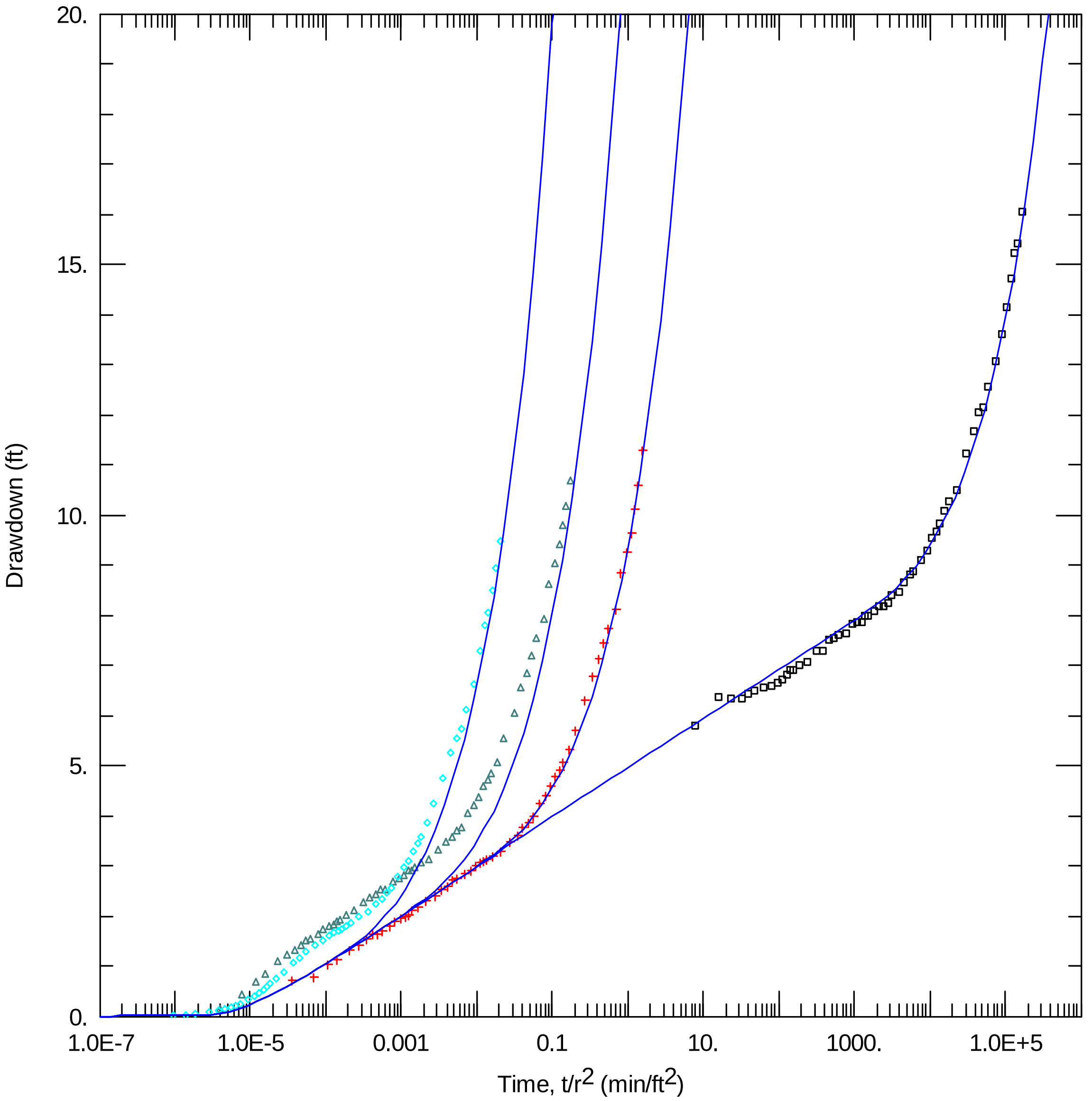

Figure 14. Buried-valley aquifer analysis of Estevan pumping test.

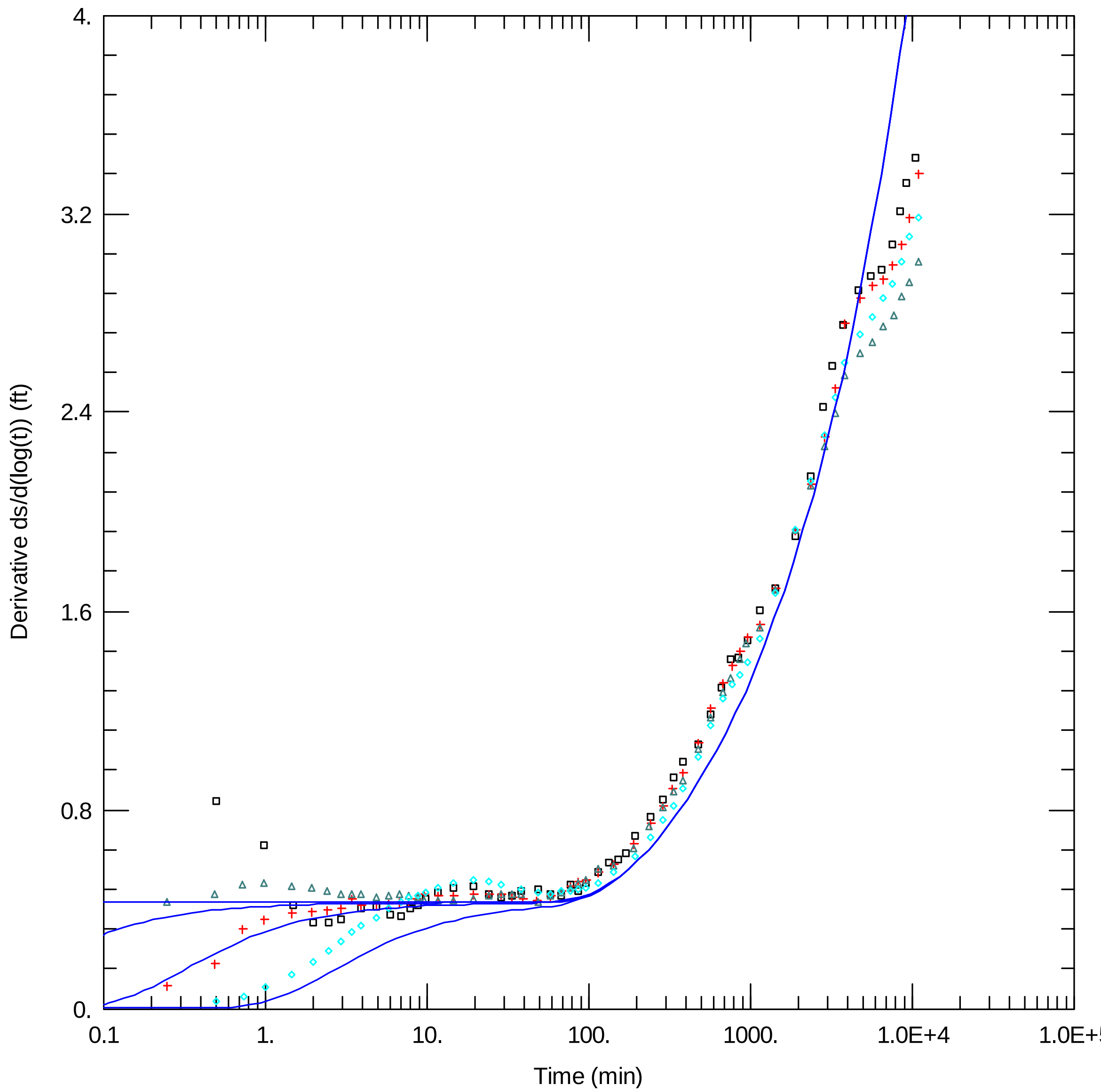

Figure 15. Drawdown Derivatives for buried-valley aquifer analysis.

6. Estimation Of The Long-Term Yield Of The Estevan Pumping Well

In this section, the long-term yield of the Estevan well is estimated using two methods:

- The Farvolden Q_{20} Method (Farvolden, 1959); and
- The Modified Moell Method (Maathuis and van der Kamp, 2006).

It has been recommended previously that the Farvolden Q_{20} Method be abandoned for estimating the long-term yield of a well. The Estevan case study is used to highlight the significant errors that may be introduced when the Q_{20} is used for a typical application in the Western Glaciated Plains.

6.1 Estimation of long-term safe pumping rate using the Farvolden Q_{20} Method

The Farvolden Q_{20} Method is developed from the extrapolation of the Cooper-Jacob straight-line analysis to 20 years of pumping. It is assumed implicitly that the drawdowns in the pumping well approximate a straight-line when plotted against the logarithm of time. The semi-log plot of the pumping well drawdowns along with the Cooper-Jacob straight-line analysis are presented in Figure 16. It is clear from the plot of the pumping well drawdowns that the aquifer does not respond as an ideal "Theis" aquifer. In particular, the drawdowns accelerate after about 100 minutes. It is important to recognize immediately that extending the straight-line portion of the response is likely to significantly underestimate the drawdowns after 20 years of pumping.

The safe yield calculated with the Farvolden Q_{20} Method is given by:

$$Q_{20} = 0.7 \times 0.68\, T\, H_A$$

where T is the transmissivity and H_A is the allowable drawdown after 20 years of pumping.

The Cooper-Jacob straight-line analysis shown in Figure 16 yields a transmissivity of 18,400 ft^2/d (1,710 m^2/d). According to Maathuis and van der Kamp (2006; p. 35), the allowable drawdown at the location of the Estevan 1965 pumping test is 73 m. Substituting the values for T and H_A into the Q_{20} calculation yields:

$$\begin{aligned} Q_{20} &= 0.7 \times 0.68\, T\, H_A \\ &= 0.7 \times 0.68 \left(1,710\ \mathrm{m^2/d}\right)\left(73\ \mathrm{m}\right) \\ &= 59,400\ \mathrm{m^3/d} \end{aligned}$$

Maathuis and van der Kamp (2006; p. 35) report a higher estimate of 97,700 m^3/d. In their calculation, they used the transmissivity of 2,800 m^2/d obtained by Walton (1970) from a Theis type-curve analysis of the first 20 minutes of the drawdowns at Observation Well #3, located 220 m from the production well.

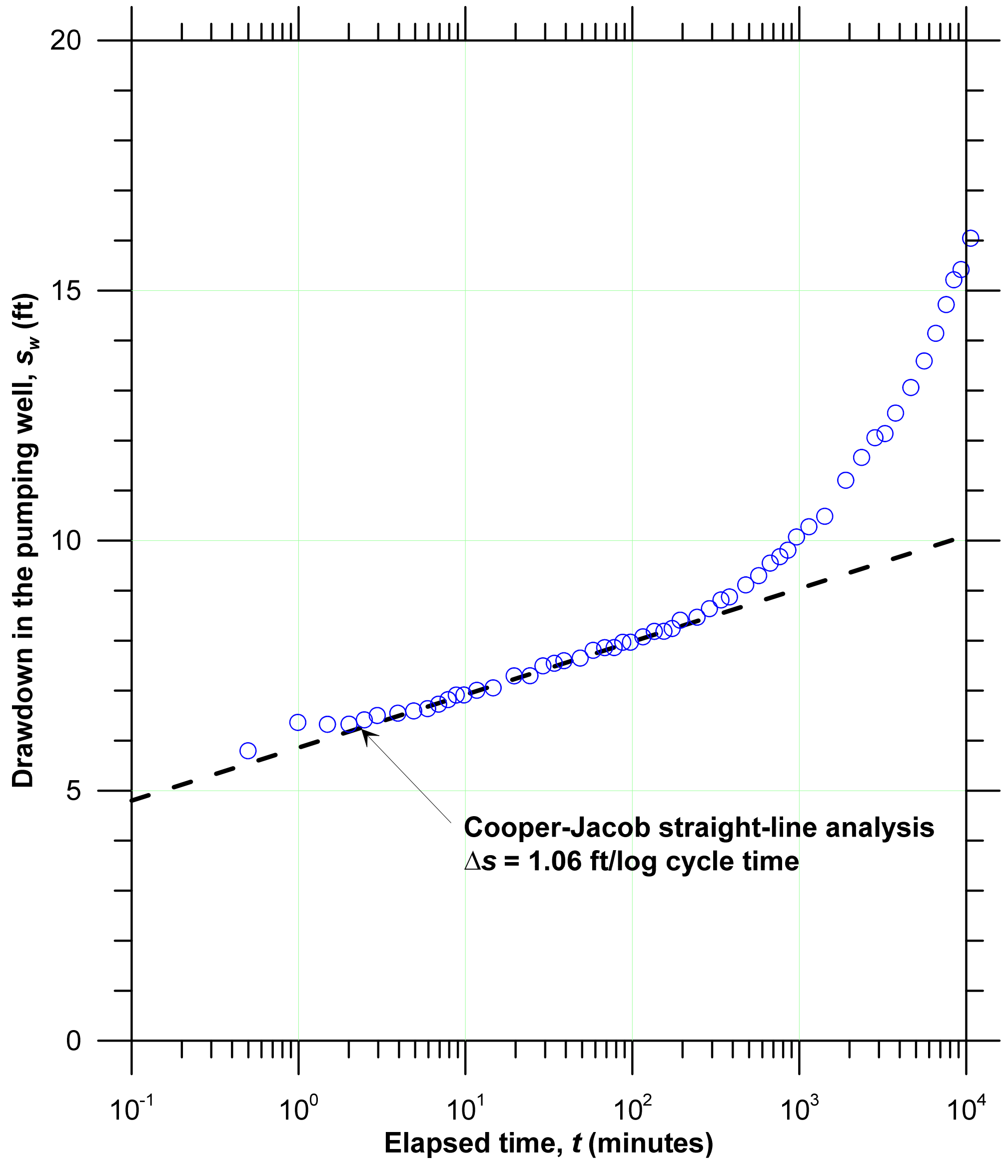

Figure 16. Pumping well drawdowns with Cooper -Jacob straight-line analysis.

6.2 Estimation of safe yield using the Modified Moell Method

The safe yield is calculated with the Modified Moell Method (Maathuis and van der Kamp, 2006) according to:

$$Q_{20} = 0.7 \times \frac{Q\,H_A}{s_{100\,\mathrm{min}} + \left(s_{20\,\mathrm{yrs}} - s_{100\,\mathrm{min}}\right)_{\mathrm{theo}}}$$

The following definitions are recalled:

- Q_{20} Sustainable yield for 20 years of pumping (m^3/d);
- Q Actual discharge rate during the pumping test (m^3/d);
- H_A Available drawdown (m);
- $s_{100\ min}$ Measured drawdown after 100 minutes of pumping (m);
- $s_{100\ min\text{-}theo}$ Calculated theoretical drawdown after 100 minutes of pumping (m); and
- $s_{20\ yrs\text{-}theo}$ Calculated theoretical drawdown after 20 years of pumping (m).

The *Alberta Environment Guide to Groundwater Authorization* (March 2011) does not provide specific guidance on the estimation of the theoretical drawdowns for the Modified Moell Method. It is indicated that the use of the Modified Moell Method must be consistent with the appropriate aquifer model. Rationale for the chosen aquifer model must be provided with supporting data (p. 13).

For this case study, the conceptual model inferred from the aquifer test analysis is used to predict the drawdowns after 20 years of pumping. The model of a buried-valley aquifer yielded an excellent match to all of the observed drawdowns for the 11,520 minutes of the test (8 days). It is important to note that regardless of the particular method used to estimate the long-term yield of the well, the estimation generally involves significant extrapolation. This is highlighted in Figure 17, in which the drawdowns observed in the pumping well are plotted with a time axis that extends to 20 years of pumping.

It is also important to note that the model of a buried-valley aquifer that has been adopted is relatively simple. The conceptual model is sufficiently simple that an analytical approach is still tractable, but the model nevertheless captures the essential elements of a buried channel aquifer system. The structure of typical buried-channel aquifer in a Canadian Prairie setting may be more complex, and under those circumstances the appropriate approach for estimating the long-term yield of a production well may involve the development of a numerical model. Indeed, at least two groundwater flow models have been developed to assess the long-term sustainability of groundwater resources in the Estevan area (Walton, 1970; p. 543; van der Kamp, 1985).

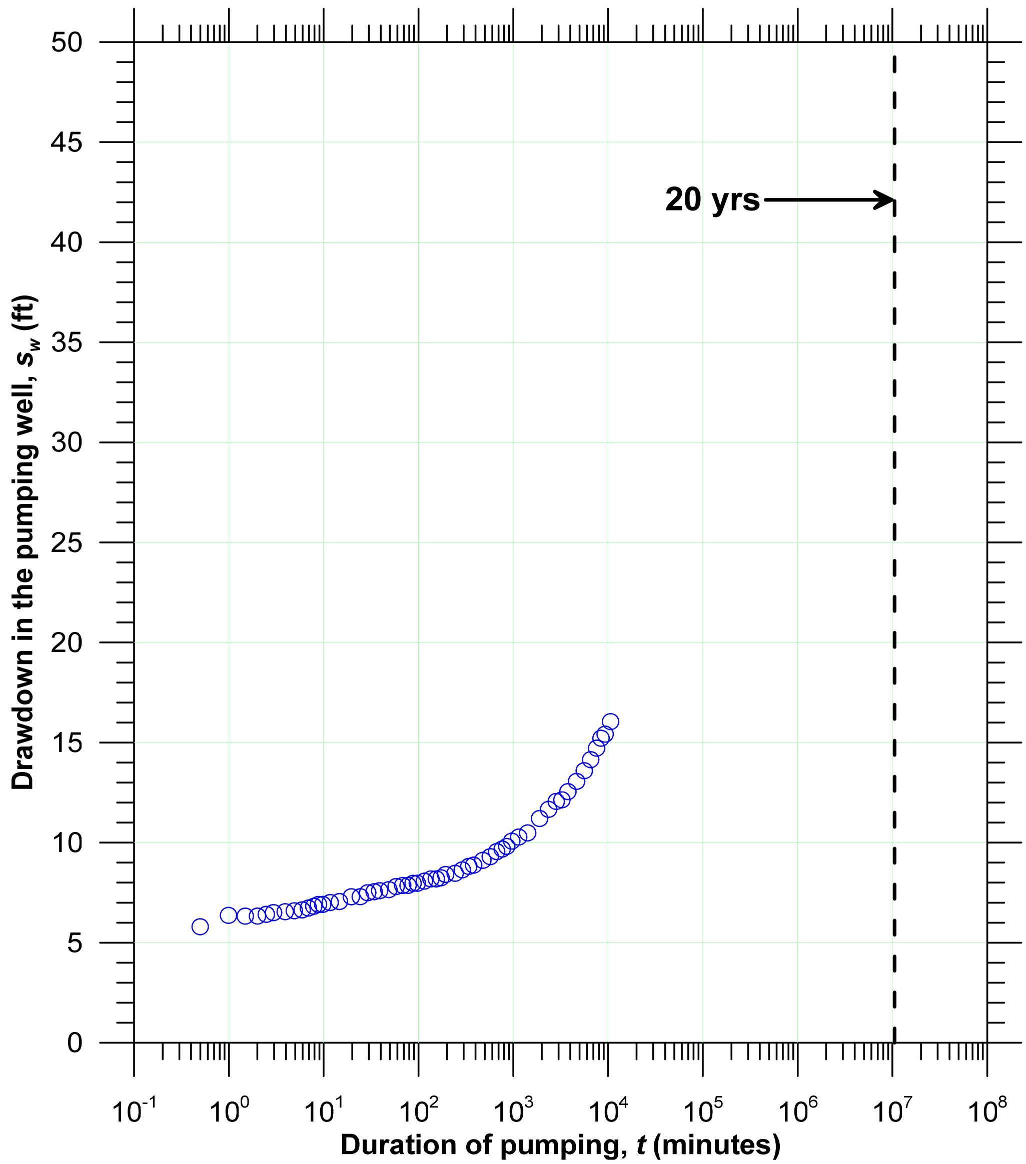

Figure 17. Pumping well drawdowns with time extended to 20 years.

The drawdowns in the pumping well predicted after 20 years with the buried-valley model are plotted in Figure 18. In this plot, the drawdowns extend beyond the limits of the drawdown axis. To accommodate the entire range of drawdowns, the results are re-plotted in Figure 19 on log-log axes. As shown in the figure, a theoretical drawdown of 278 ft (84.7 m) is predicted after 20 years of pumping. This exceeds the allowable drawdown of 73 m, suggesting that the pumping rate during the Estevan test is not sustainable over the long term.

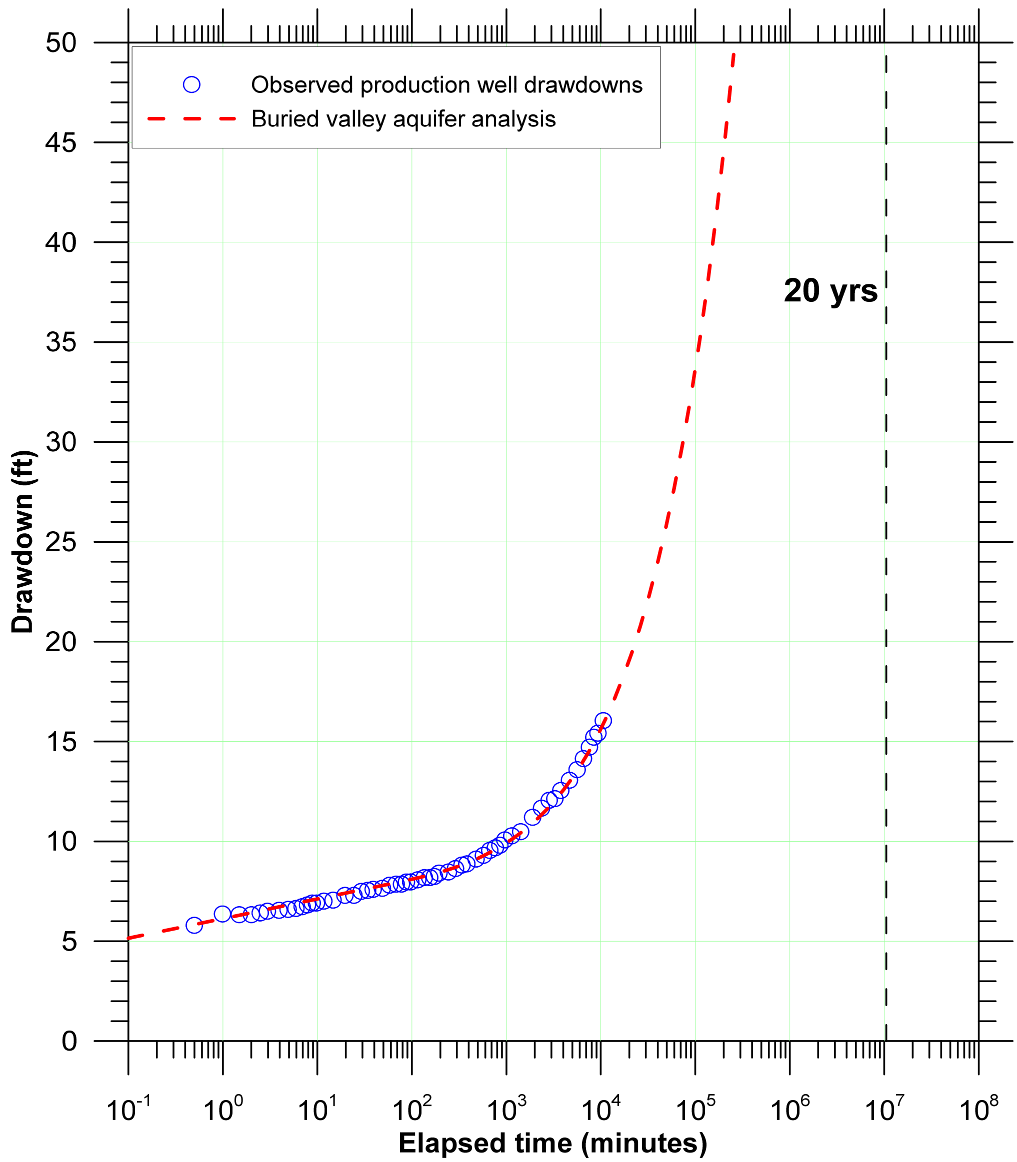

Figure 18. Long-term pumping well drawdowns predicted with the buried-valley model.

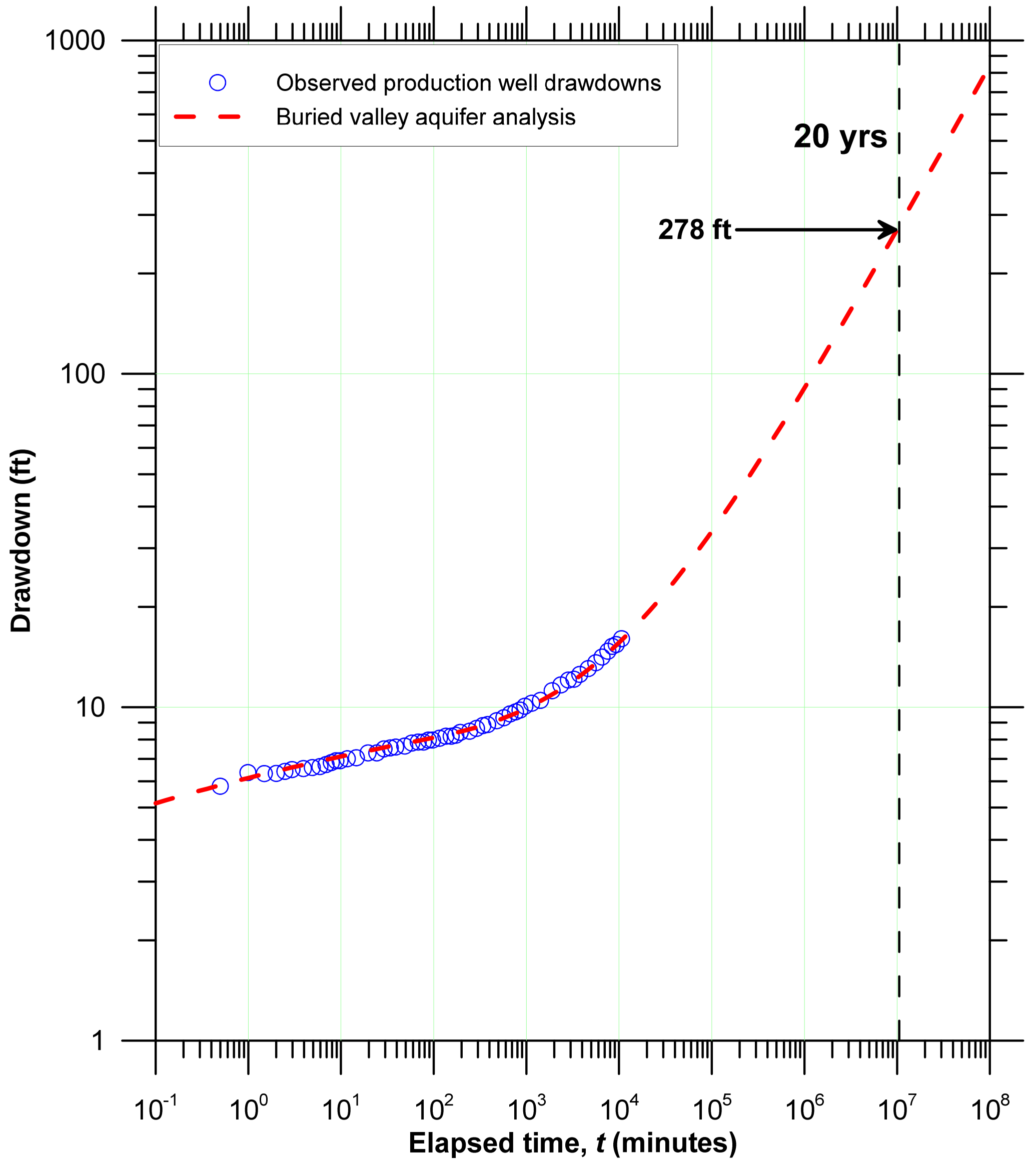

Figure 19. Long-term pumping well drawdowns predicted with the buried-valley model, logarithmic drawdown axis.

The final step in the re-analysis of the buried-valley aquifer model is to assemble the relevant drawdowns. The match to the pumping well drawdowns was obtained without invoking any additional drawdown processes; that is, all drawdowns in the pumping well are attributed to head losses in the formation. Therefore, the observed and theoretical drawdowns at 100 minutes are almost identical. The drawdowns for input to the Modified Moell Method are:

$$s_{100\text{ min}} = 7.97\text{ ft} = 2.43\text{ m}$$
$$s_{100\text{ min-theo}} = 7.99\text{ ft} = 2.44\text{ m}$$
$$s_{20\text{ years-theo}} = 278\text{ ft} = 84.73\text{ m}$$

Recalling that the allowable drawdown at the location of the Estevan 1965 pumping test is 73 m, the Modified Moell calculation yields:

$$\begin{aligned} Q_{20} &= 0.7 \times \frac{Q\,H_A}{s_{100\text{ min}} + \left(s_{20\text{ yrs}} - s_{100\text{ min}}\right)_{\text{theo}}} \\ &= 0.7 \times \frac{(460\text{ Igpm})(73\text{ m})}{(2.43\text{ m}) + \left[(84.73\text{ m}) - (2.44\text{ m})\right]} \left| \frac{\text{m}^3}{219.97\text{ Igal}} \right| \frac{1440\text{ min}}{\text{d}} \right| \\ &= 1{,}820\text{ m}^3/\text{d} \end{aligned}$$

7. Assessment of the estimates of the long-term yield of the Estevan well

The safe yield of the Estevan well estimated with the Modified Moell Method is a relatively small fraction of the estimate developed with the Farvolden Q_{20} Method (1,820 m^3/d vs. 59,400 m^3/d). Is this reduced prediction of the safe yield realistic?

It has been argued here that the Modified Moell Method provides the most defensible estimate of the long-term sustainable yield of the Estevan well. The method is based on a conceptual model of the aquifer that is physically plausible and consistent with the hydrogeologic setting. The implementation of the conceptual model matches the complete drawdown and Drawdown Derivative records for both the production well and the three observation wells.

Maathuis and van der Kamp (2003) reported the results of an analysis they conducted in 1998, in which they evaluated the potential yield of the Estevan aquifer system based on the monitoring of drawdowns after 6 years of pumping followed by 3 years of recovery. They predicted that the sustainable yield of the Estevan valley aquifer ranged from 2,400 to 2,800 dam^3/yr (a dam^3 is 1,000 m^3). This corresponds to a pumping rate of between 6,600 to 7,600 m^3/d. This range exceeds the estimate developed here with Modified Moell Method, but is significantly lower than the estimate developed with the Farvolden Q_{20} Method.